UNIT 1 (Relativity)

Syllabus (Unit I)

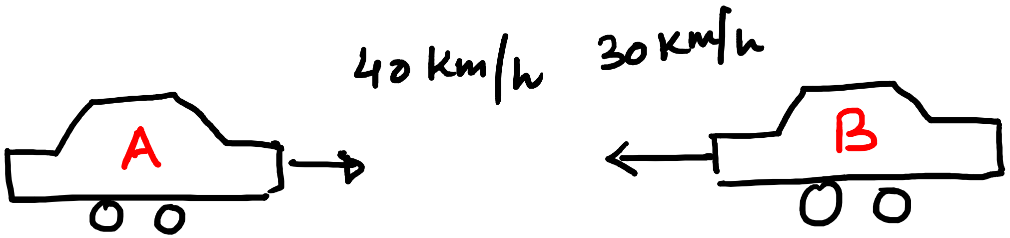
Relativistic Mechanics: Frame of reference, Inertial & non-inertial frames, Galilean transformations, Michelson- Morley experiment, Postulates of special theory of relativity, Lorentz transformations, Length contraction, Time dilation, Velocity addition theorem, Variation of mass with velocity, Einstein"s mass energy relation, Relativistic relation between energy and momentum, Massless particle.

Reference Books

→ very good book

1. Concepts of Modern Physics – Aurthur Beiser (McGraw Hill) (soft copy)
2. Introduction to Special Theory of Relativity- Robert Resnick (Wiley)
3. Optics – Brijlal & Subramanian (S. Chand)
4. Engineering Physics: Theory and Practical- Katiyar and Pandey (Wiley India)
5. Applied Physics for Engineers- Neeraj Mehta (PHI Learning, New)
6. Engineering Physics-Malik HK and Singh AK (McGrawHill)

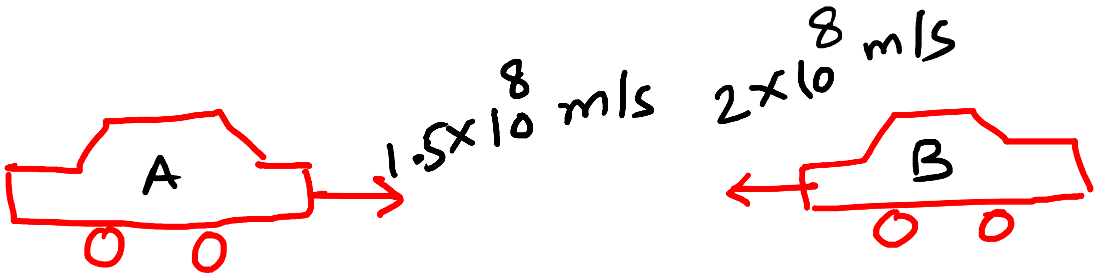
-: Relativity :-



Q:- what is the velocity of B wrt A.?

Sol^n:-

$$v_{AB} = 70\ km/h$$



Q:- what is the velocity B wrt A ?

Soln:-

$$v_{AB} = 3.5\times10^8 \text{ m/s} > c = 3\times10^8 \text{ m/s}$$

This is incorrect

Frame of Reference :-

* Absolute rest and absolute motion are not defined.
  why ?

* Rest and motion are relative.

* Space is homogeneous i.e. we can choose any point in space as origin.

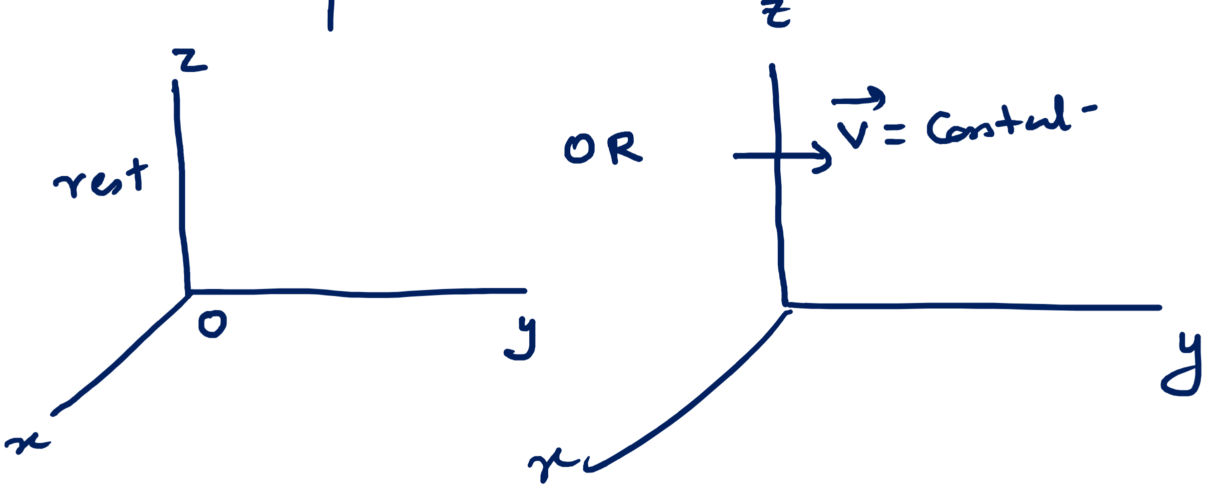
* Space is isotropic i.e. we can choose any direction in space as coordinate axis.

Types of frames :-

On the basis of applicability of Newton's law there are two types of frame of references.

1. Inertial frames :-

Unaccelerated frames are called as inertial frames. Newton's laws are valid in inertial frames.



$$\boxed{\vec{F} = \frac{d\vec{p}}{dt}}$$

2- Non-inertial Frame :-

Accelerated frames are called as non-inertial frames.

eg → Earth is an accelerating body so every frame associated with earth is a non-inertial frames.

* In non-inertial frames, Newton's laws are not valid.

$$\vec{F} \neq \frac{d\vec{p}}{dt}$$

* We can use Newton's law in non-inertial

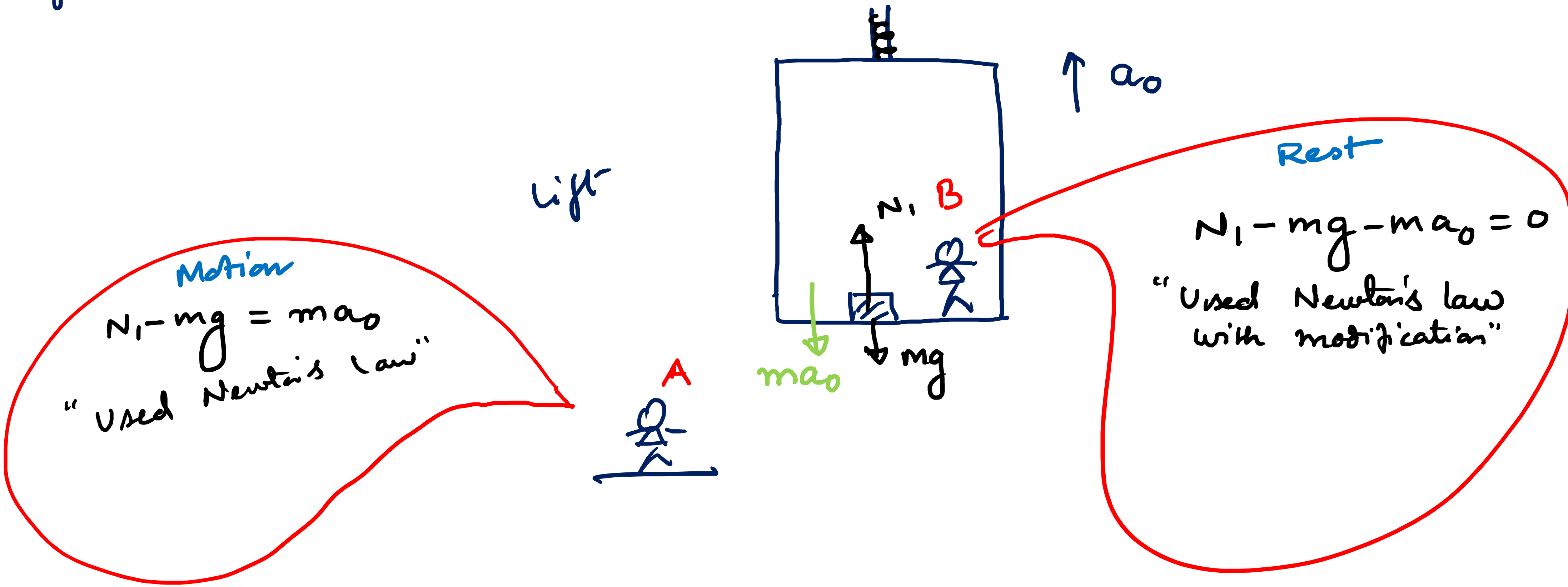
frame but apply some modifications.

$$\vec{F}_{ext}^{modi} = \frac{d\vec{p}}{dt}$$

These modifications are called as pseudo forces.

How can we say that Newton's laws are not valid in non-inertial frame?

Explanation :-



See you in next class

What is the nature of light ??

1. Newton's particle theory of light :-

$$v_{denser} > v_{rarer}$$

2- Huygen's wave theory of light :-

* wave front-
* secondary wavelet
* Hypothetical medium "Aether" or "ether" → Heavenly air.

$$v_{denser} < v_{rarer}$$

3- Focault :-

$$v_{denser} < v_{rarer}$$

4. Maxwell's EMT of light :-

$$\left\{ \begin{array}{l} \nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}, \quad \nabla \cdot \vec{B} = 0, \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \end{array} \right.$$

light waves are Electromagnetic waves.

5- Still photoelectric effect, compton's effect, Hydrogen spectrum etc. can not be explained by wave theory.

6- Plank's quantum theory :-

$$\boxed{E = nh\nu} \qquad n = 1, 2, 3 \ldots$$

7. Albert Einstein (in 1905) explained photoelectric effect with the help of quantum theory of light.

Answer

"Light has dual nature"

8. De. Broglie's matter waves:

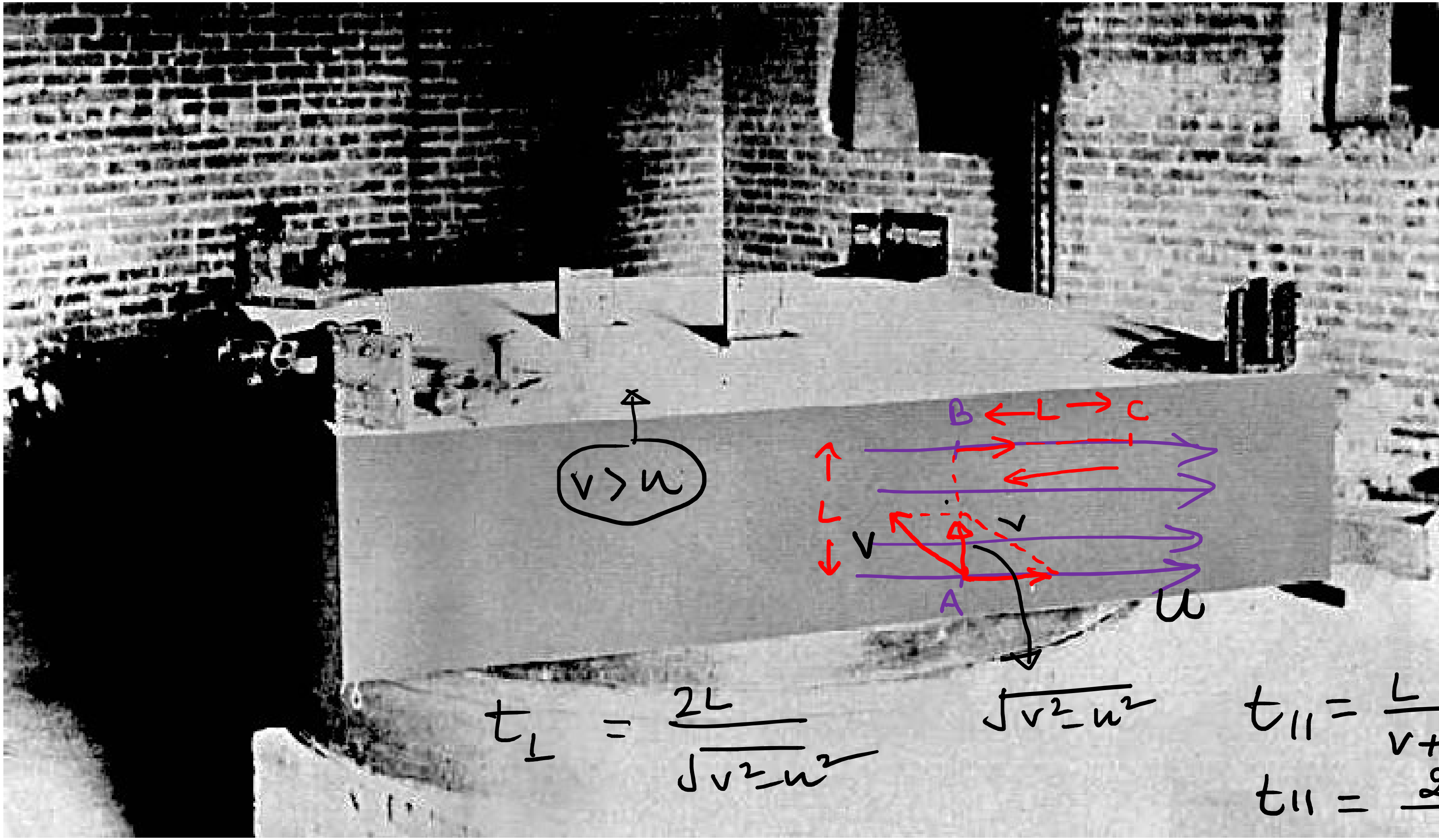
$$\lambda = \frac{h}{p}$$

9. Schroedinger's eqn :-

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

ψ → variable in matter wave

Michelson–Morley experiment



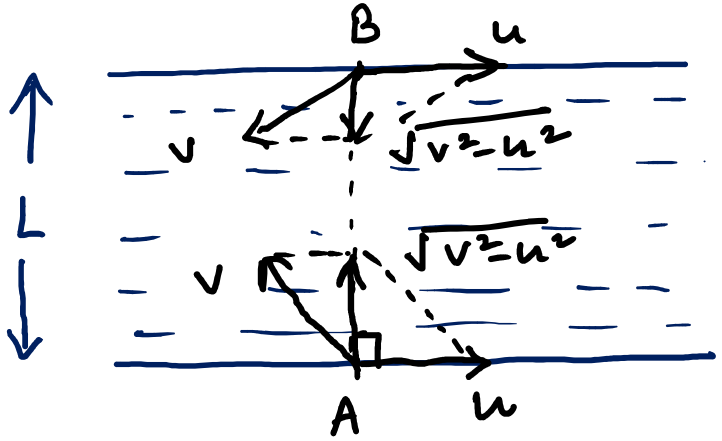
$$t_{\perp} = \frac{2L}{\sqrt{v^2-u^2}}$$

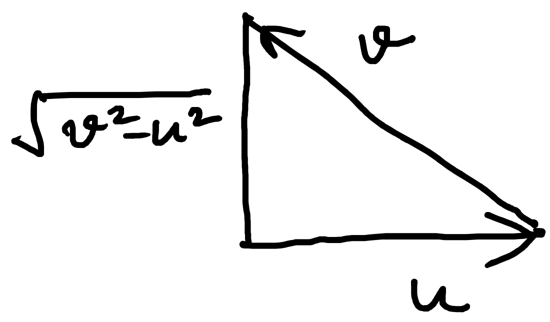
$$t_{\parallel} = \frac{L}{v+u} + \frac{L}{v-u}$$

$$t_{\parallel} = \frac{2Lv}{v^2-u^2}$$

River Swimmer problem :-

Consider the speed of river is u and speed of swimmer is v.





⇒ Transverse motion :-

$$t_\perp = t_{AB} + t_{BA} = \frac{L}{\sqrt{v^2-u^2}} + \frac{L}{\sqrt{v^2-u^2}}$$

$$t_\perp = \frac{2L}{\sqrt{v^2-u^2}} = \frac{2L}{v\sqrt{1-\frac{u^2}{v^2}}}$$

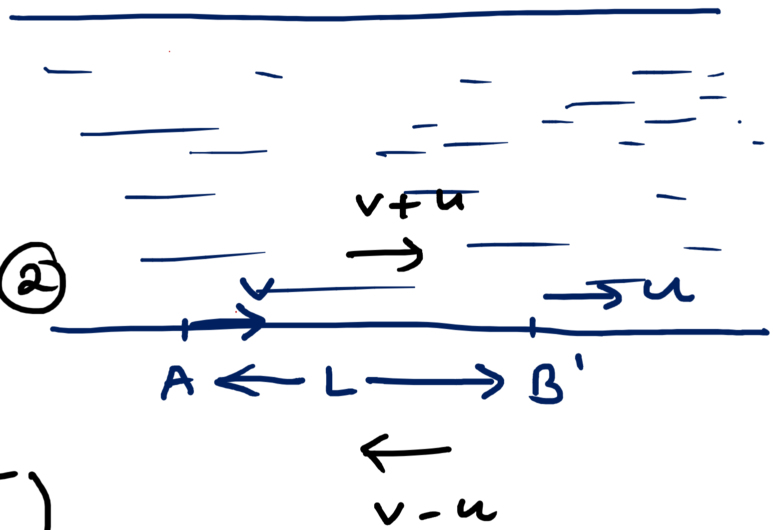
$$t_\perp = \frac{2L}{v}\left[1+\frac{u^2}{2v^2}\right]$$

$\Rightarrow$ longitudinal motion:

$$(1+x)^n \approx 1 + nx$$

$$t_{\parallel} = t_{AB'} + t_{B'A}$$

$$= \frac{L}{v+u} + \frac{L}{v-u}$$



$$t_{\parallel} = \frac{2vL}{v^2-u^2} = \frac{2L}{v\left(1-\frac{u^2}{v^2}\right)} \quad — (2)$$

$$t_{\parallel} \approx \frac{2L}{v} \cdot \left(1 + \frac{u^2}{v^2}\right)$$

$$t_{\perp} \approx \frac{2L}{v}\left(1 + \frac{u^2}{2v^2}\right)$$

$$\Delta t = t_{\parallel} - t_{\perp} = \frac{2L}{v} \cdot \frac{u^2}{2v^2} = \boxed{\frac{Lu^2}{v^3}}$$

$$\Delta t \approx \frac{L u^2}{v^3}$$

M.M. Imp

M. M experiment :-

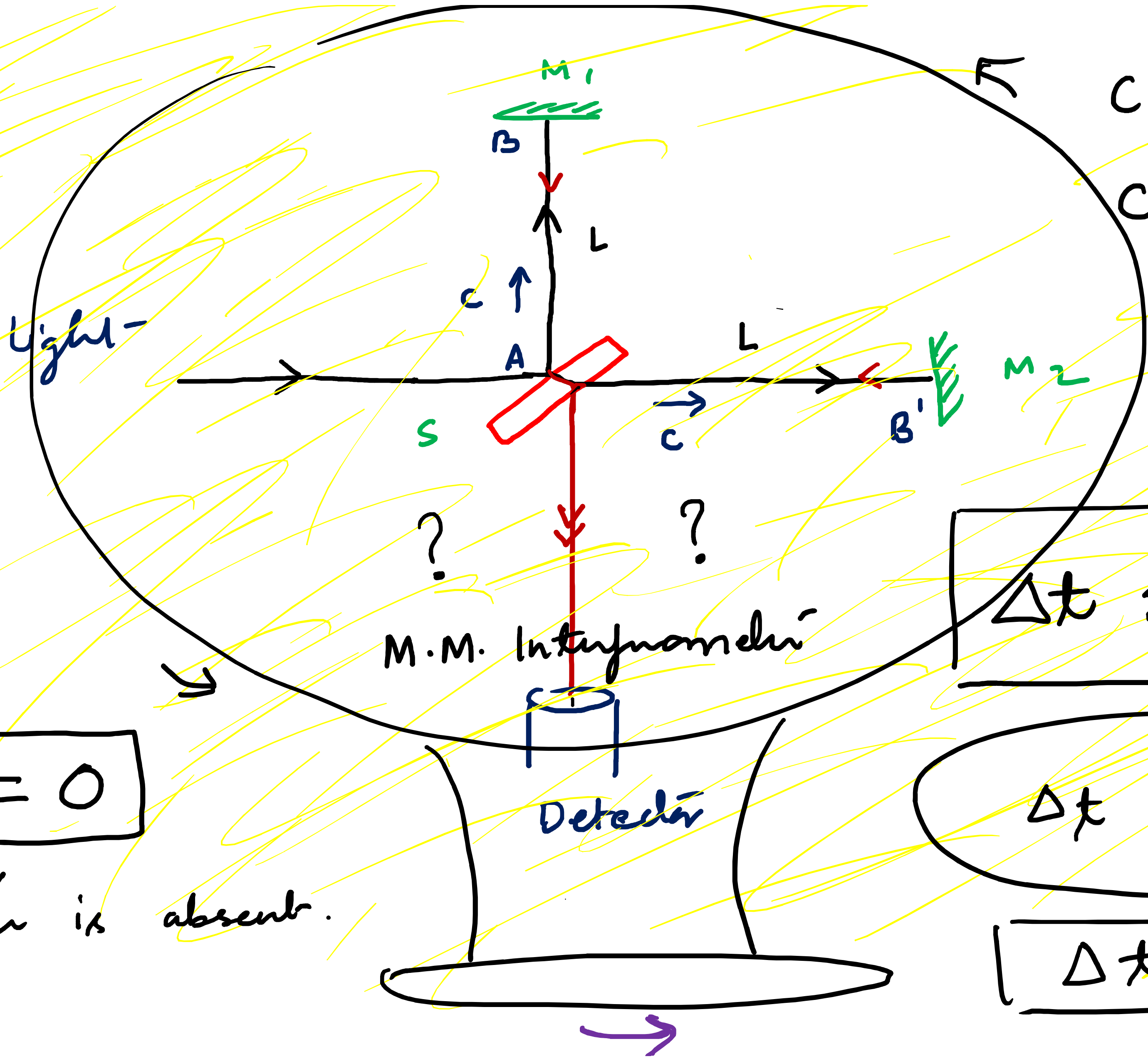
Swimmer	$\longleftrightarrow$	photon or light
River banks	$\longleftrightarrow$	Mirrors
River	$\longleftrightarrow$	Aether (or ether)

→ History

$C = 3 \times 10^8 \ m/s$

or

$C = 300000 \ km/sec$



$$\Delta t \approx \frac{L u^2}{c^3}$$

$$\Delta t \approx 10^{-24} \ sec$$

$$\Delta t = 0$$

↓

Aether is absent.

$$\Delta t \approx 0$$ → Aether is present

Nature of light

particle

(Newton)

wave (Aether or ether)

(Huygens)

In English,

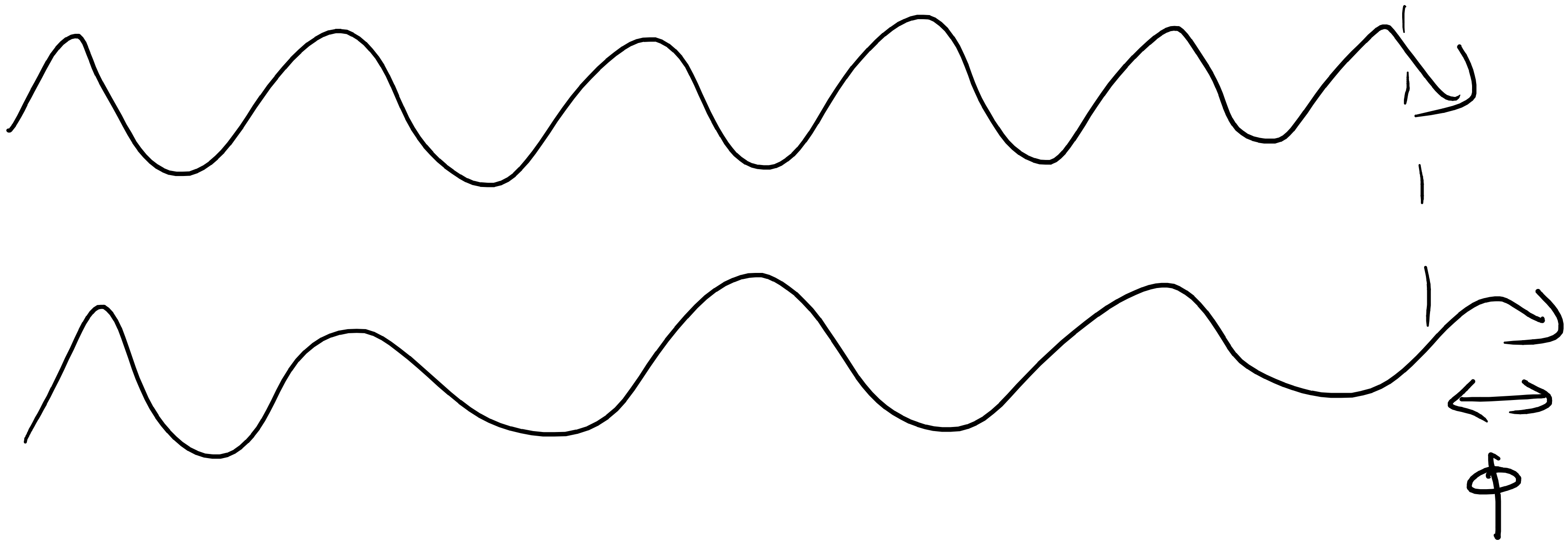
"Heavenly Air"

M.M. experiment

Nobel price

①	②
$\Delta t = 0$	$\Delta t \approx 0$
↙	↘
Aether is not present	Aether is present
✓ Result	

1970 → Result

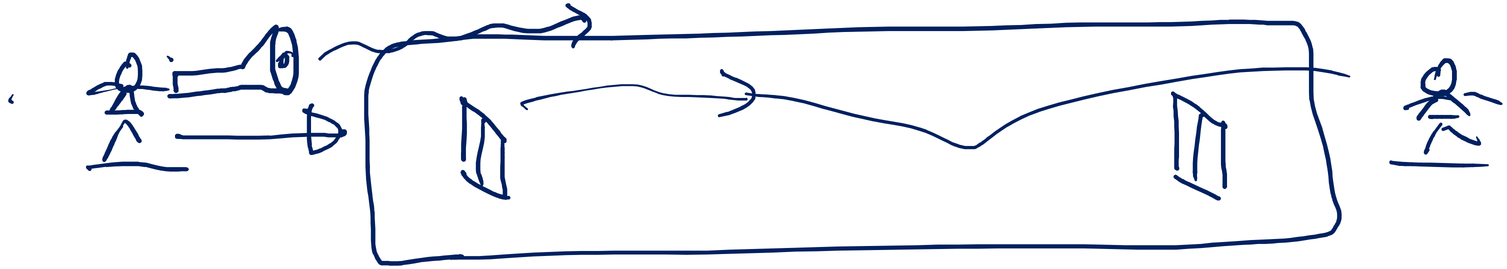

Φ


SOLVAY CONFERENCE (BRUSSELS 1927)
A. Piccard | E. Henriot | P. Ehrenfest | E. Herzen | Th. de Donder | E. Schrödinger
J.E. Verschaffelt | W. Pauli | W. Heisenberg | R.H. Fowler | L. Brillouin
P. Debye | M. Knudsen | W.L. Bragg | H.A. Kramers | P.A.M. Dirac
A.H. Compton | L. de Broglie | M. Born | N. Bohr
I. Langmuir | M. Planck | M. Curie | H.A. Lorentz | A. Einstein
P. Langevin | Ch.-E. Guye | C.T.R. Wilson | O.W. Richardson
Original black and white image colorized by PastInColour.com

Results of M.M. exp. :

* There is no ether or aether type medium.
* Velocity of light in vaccume i.e c is irrespective. or velocity of light in vaccum is absolute.

$\not{c} + V = C$



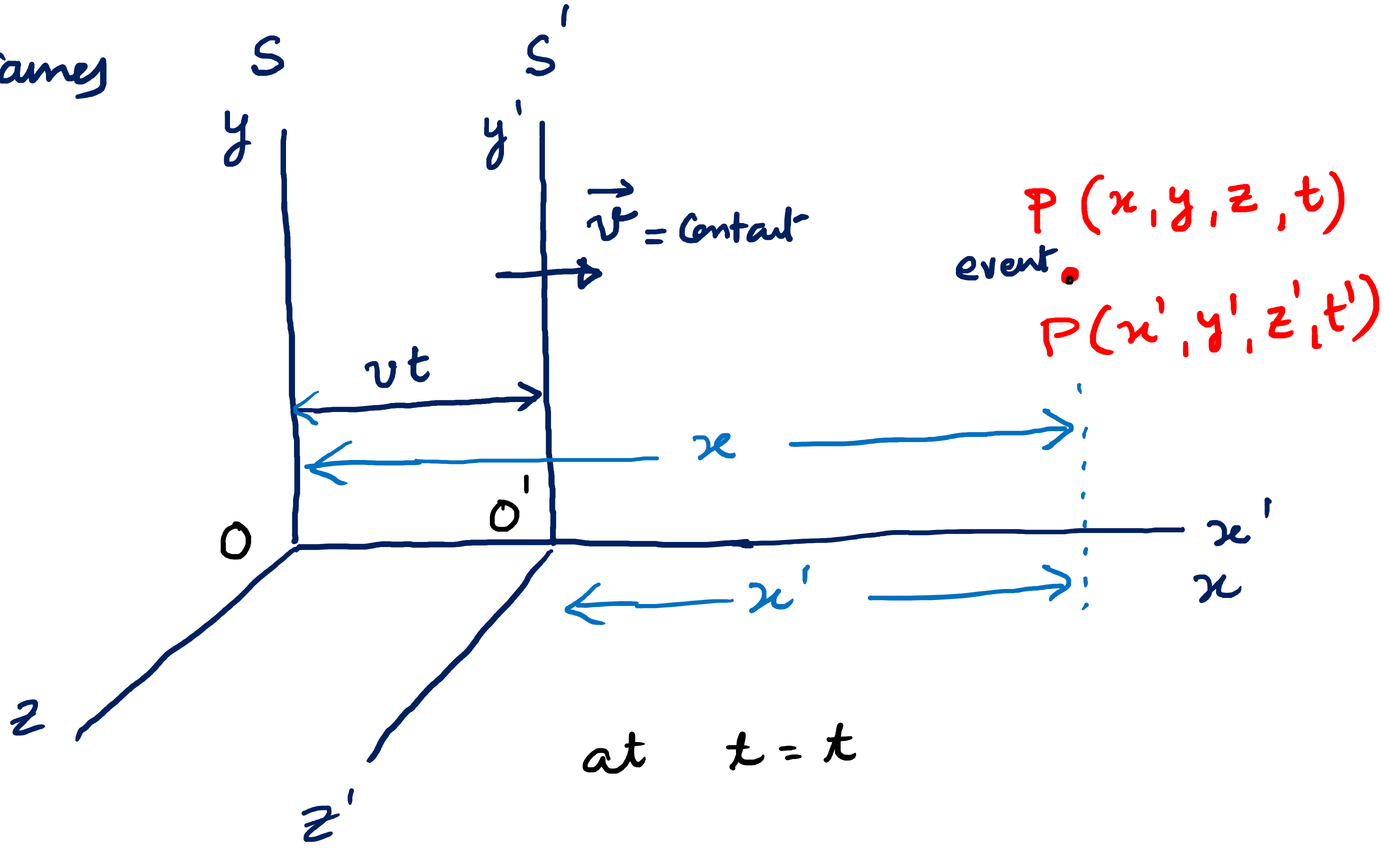
-: Kinematics :-

Galilean Transformation :- (Check point)

There are two inertial frames S and S'.

G.T. eq^n.

$$\begin{cases} x' = x - vt \\ y' = y \\ z' = z \\ t' = t \end{cases}$$



In general,

$$\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$$

In that case

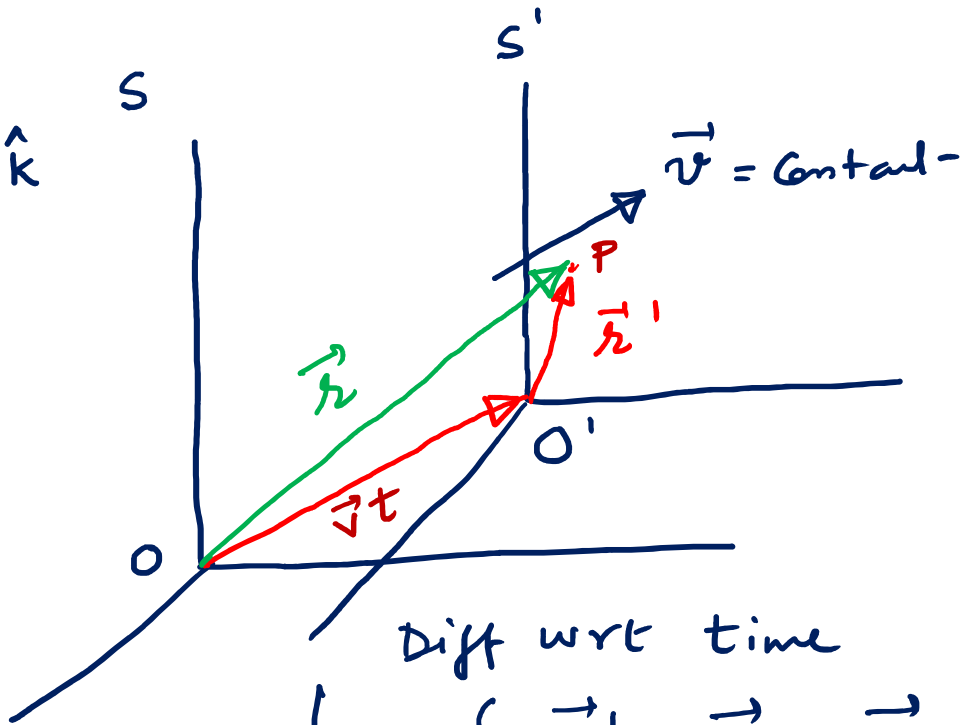
G.T. eq^n

$$\left\{ \begin{aligned} x' &= x - v_x t \quad \times \hat{i} \\ y' &= y - v_y t \quad \times \hat{j} \\ z' &= z - v_z t \quad \times \hat{k} \\ t' &= t \end{aligned} \right.$$

OR

G.T. eq^n

$$\left\{ \begin{aligned} \vec{r}\,' &= \vec{r} - \vec{v}\, t \quad \text{for space} \\ t' &= t \quad \text{for time} \end{aligned} \right.$$



Diff wrt time

G.T. eq^n $\left\{ \vec{u}\,' = \vec{u} - \vec{v} \right.$ for vel

Again diff. wrt time

G.T. eq^n $\left\{ \vec{a}\,' = \vec{a} \right.$ for acc.

G.T. eq^n $\left\{ \vec{F}\,' = \vec{F} \right.$ for force.

- Maxwell’s equations are nice and the speed of light is *c* was called the frame of the *ether*.
- Maxwell’s equations were non-invariance under the Galilean transformations)

$\nabla \cdot \vec{B} = 0$ or $\oint \vec{B} \cdot d\vec{s} = 0$

- There were therefore two possibilities.

  - Either something was wrong with Maxwell’s equations,

    or

  - Something was wrong with the Galilean transformations.

- Einstein **finally** showed that the trouble was in fact with Galilean transformations.

- More precisely, in 1905 he showed that the Galilean transformations are valid only when the speed involved is much less than the speed of light.5

MIRACLE YEAR 1905

Four papers of **Albert Einstein** published in the in 1905.

1. Photoelectric effect → Nobel 12th
2. Brownian motion → BSc.
3. Special relativity mechanics
4. Mass–energy equivalence ($E = mc^2$)

B. Tech.

Fundamental postulates of STR :- Imp

i) The speed of light in vaccume is same in all inertial frame.

* c is invariant.
* c is irrespective.
* c is absolute.

ii) All inertial frames are equivalent.

It also says that

* space is homongeneous and isotropic.
* Absolute rest and absolute motion are not defined.

* laws of physics are hold in all inertial frames.

* Time dilation
* Loss of sim
* Length Contraction

} → Lorentz's Transformation (True)

"Thank you"

M.M. Imp

∴ Lorentz Transformation :- (True transformation)

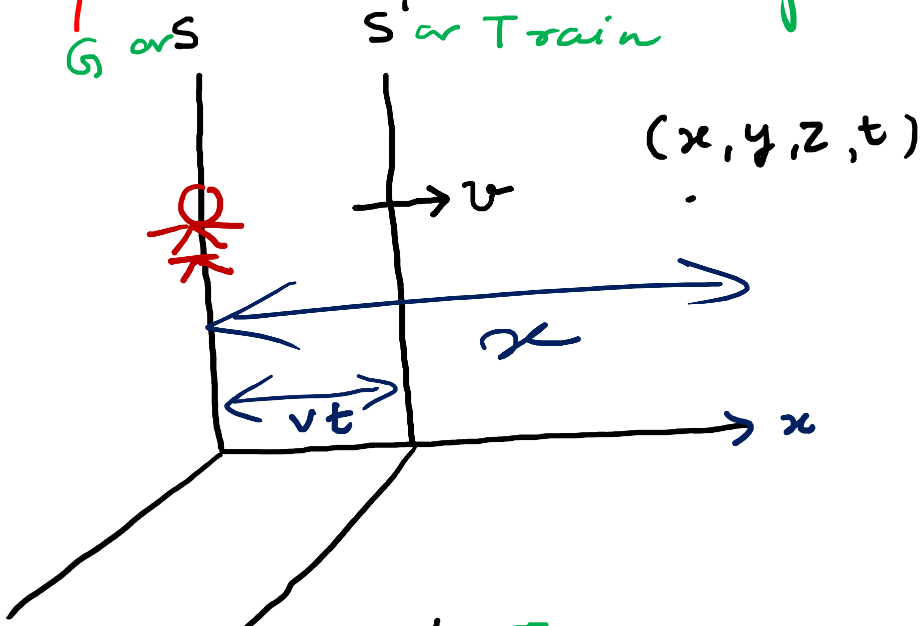
According to Lorentz,

$$x' \propto x - vt$$

$$\Rightarrow x' = A(x - vt) \quad \text{--- (1)}$$

$$y' = y$$

$$z' = z$$



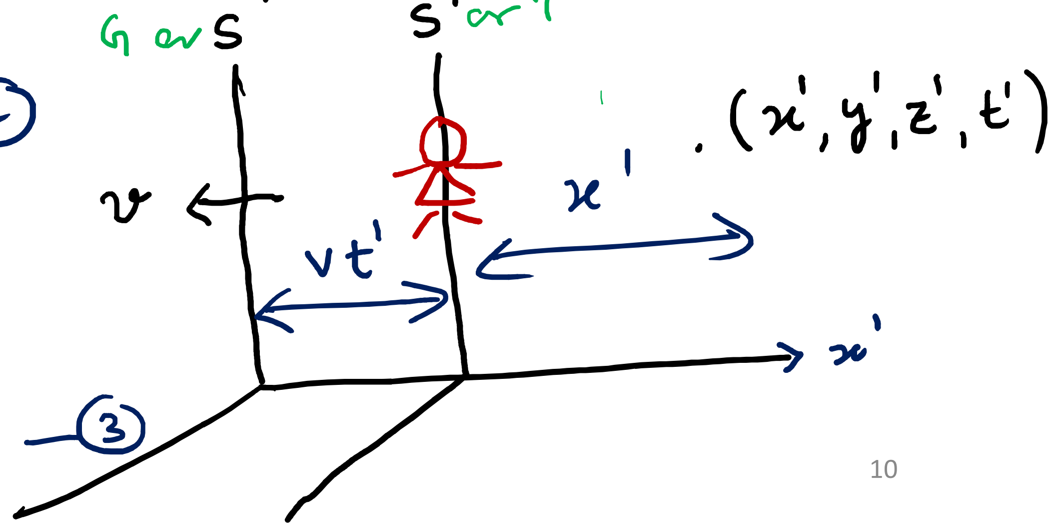
Post-1 Since all frames are eqt.,

$$x = A(x' + vt') \quad \text{--- (2)}$$

From eqn (1) and (2)

$$x = A[A(x - vt) + vt']$$

$$x = A^2 x - A^2 vt + Avt' \quad \text{--- (3)}$$



Post:- Since c is same in all inertial frame

$$\frac{x}{t} = c \Rightarrow x = ct \quad \text{--- (4)}$$

$$\frac{x'}{t'} = c \Rightarrow x' = ct' \quad \text{--- (5)}$$

Using eqn (4) & (5) in eqn (3)

$$x = A^2x - A^2vt + Avt'$$

$$ct = A^2ct - A^2vt + Av\frac{x'}{c}$$

$$ct = A^2ct - A^2vt + A\frac{v}{c}.A(x - vt) \quad \text{(using eqn (1))}$$

$$ct = A^2ct - A^2vt + A^2\frac{v}{c}(ct - vt) \quad \text{(using eqn (4))}$$

$$A = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma \quad \text{(Lorentz factor)}$$

Using eq^n (1)

$$x' = A(x - vt)$$

$$ct' = \gamma(ct - vt) \qquad \text{(Using eq}^n\text{ (4) and eq}^n\text{ (5))}$$

$$t' = \gamma\left(t - \frac{vt}{c}\right)$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

Thus Lorentz Transformation eq^{n}s are

$$\left\{\begin{aligned} x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma\left(t - \frac{vx}{c^2}\right) \end{aligned}\right\}$$

called as L.T. eq^n

OR

$$\left\{\begin{aligned} x &= \gamma(x' + vt') \\ y &= y' \\ z &= z' \\ t &= \gamma\left(t' + \frac{vx'}{c^2}\right) \end{aligned}\right\}$$

called as Inverse L.T. eq^n

Note :-

$$x' = \gamma(x - vt)$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \quad \times v$$

$+ \qquad +$

$$x' + vt' = \gamma\left(x - \frac{v^2}{c^2}x\right) = \gamma x\left(1 - \frac{v^2}{c^2}\right)$$

$$x' + vt' = \gamma x \frac{1}{\gamma^2}$$

$$x = \gamma(x' + vt')$$

similarly

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

Lorentz's Transformation :-

According to Lorentz,

$$x' \propto x - vt$$

or

$$x' = \gamma(x - vt)$$

and

$$y' = y$$

$$z' = z$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

} L.T. eq^{n}s.

In units of c, (c = 1)

L.T. eqⁿ

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma(t - vx)$$

$\equiv$

Inverse L.T. eqⁿ

$$x = \gamma(x' + vt')$$
$$y = y'$$
$$z = z'$$
$$t = \gamma(t' + vx')$$

For inverse eqⁿ.

$$x' = \gamma x - \gamma v t$$
$$t' = -v\gamma x + \gamma t \qquad \times v$$

$$x' + vt' = (1 - v^2)\gamma x$$

$$x' + vt' = \frac{1}{\gamma^2}\gamma x \Rightarrow \boxed{x = \gamma(x' + vt')}$$

Similarly $\boxed{t = \gamma(t' + vx')}$

$$\begin{cases} x' = \gamma(x - \beta t) \\ y' = y \\ z' = z \\ t' = \gamma(t - \beta x) \end{cases}$$

$$\beta = \frac{v}{c} \quad . \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$(\underset{t}{x})' = \gamma\left[(\underset{t}{x}) - v(\underset{x}{t})\right]$$

Prob:-

Prove that $x^2 + y^2 + z^2 - c^2t^2$ is invariant under L.T.

Sol^n:-

$$x'^2 + y'^2 + z'^2 - c^2t'^2 = \gamma^2(x - vt)^2 + y^2 + z^2 - c^2\gamma^2\left(t - \frac{vx}{c^2}\right)^2$$

$=$

$=$ solve

$=$

$= x^2 + y^2 + z^2 - c^2t^2$

$$x^2 + y^2 + z^2 - c^2t^2 = \text{Invariant}$$

or

$$x^2 - c^2t^2 = \text{Invariant}$$

$$x^2 + y^2 + z^2 - c^2t^2 = \text{Invariant}$$

or

$$\Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2 = \text{Invariant}$$

Quantities invariant under L.T. :

* Rest mass / Rest mass energy
* $\Delta x^2 - c^2 \Delta t^2$
* charge.
* Maxwell's eq^{n}s.

* $\vec{E}^2 - c^2 \vec{B}^2$
* $\vec{E} \cdot \vec{B}$
* c

etc.

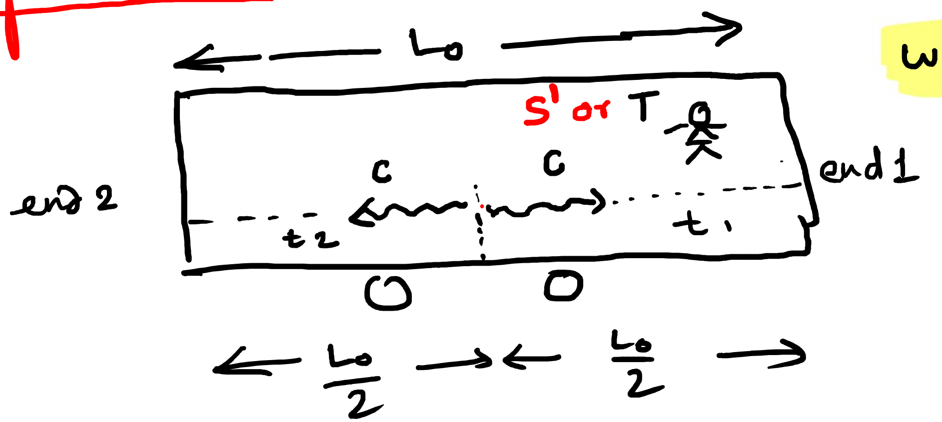
Fundamental effects of these postulates :

1. Loss of simultaneity :

if two events are simultaneous in one inertial frame S then same events not simultaneous in other moving frame S' wrt S.

$$e^+ + e^- \rightarrow h\nu + h\nu$$ (for example)

Explaination :



wrt T,

$$t_1 = t_2 = \frac{L_0/2}{c} \quad \text{--- (1)}$$

i.e. events are simultaneous

wrt G :

$$t_1' > t_2'$$

for 1st photon

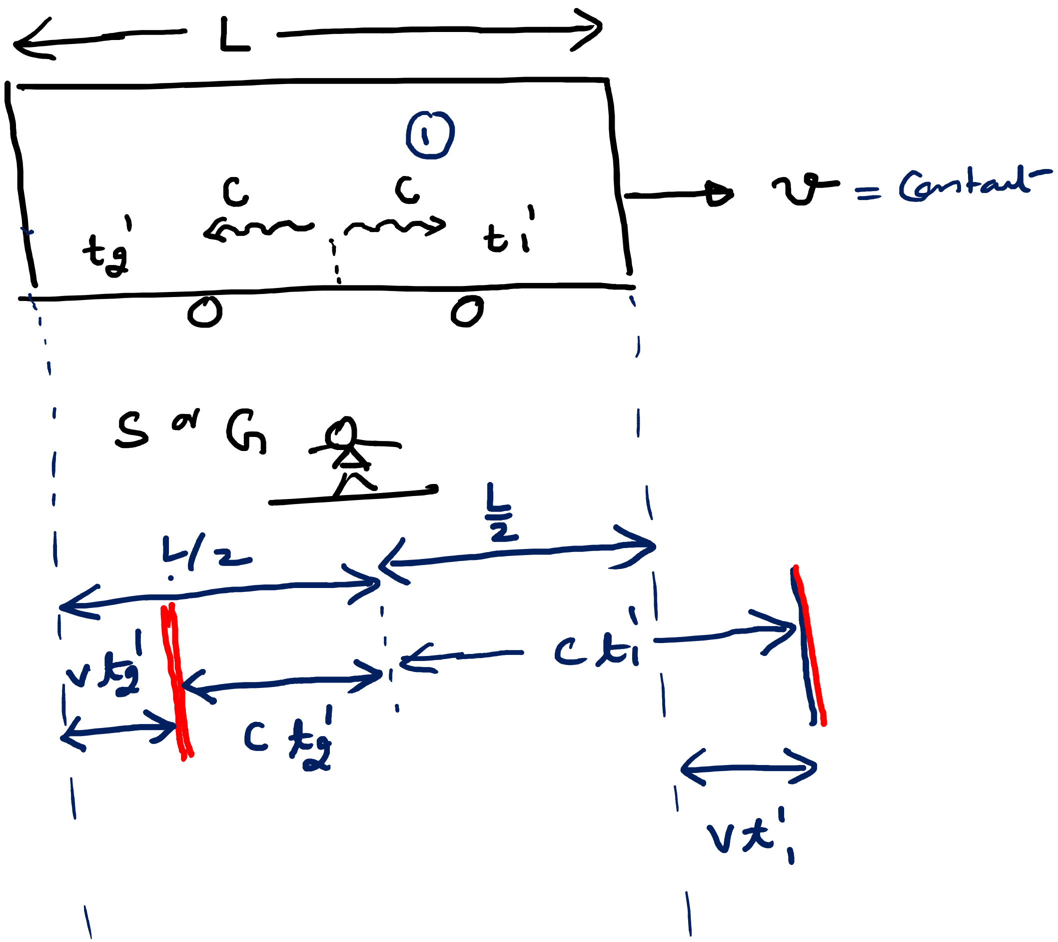
$$c t_1' - v t_1' = \frac{L}{2}$$

$$t_1' = \frac{L/2}{c-v} \qquad \text{—— (1)}$$

Similarly for IInd photon,

$$c t_2' + v t_2' = \frac{L}{2}$$

$$t_2' = \frac{L/2}{c+v} \qquad \text{—— (2)}$$



$$t_1' > t_2'$$

Same events are not simultaneous.

* if $v << c$ $t_1' = t_2'$, you can feel this fact when $v \approx c$.

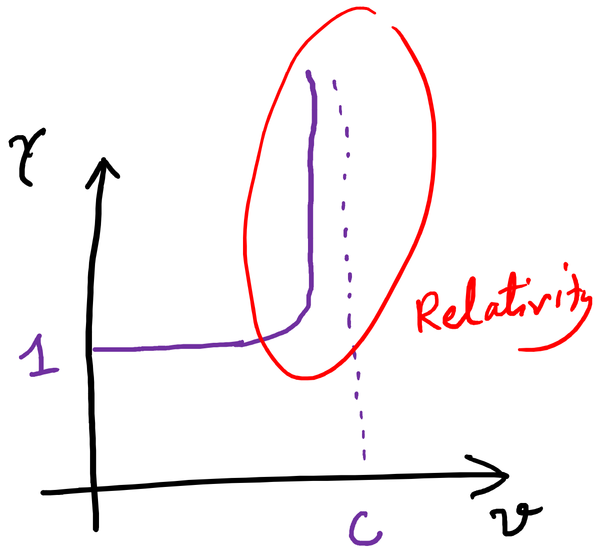
↓

Loss of Sim.

Note:-

v	γ	
0	1	non-relativistic particles
20 m/s	1	
20000 m/s	1	
10^7 m/s	1.0016	
$\frac{c}{10}$	1.0050	
0.9c	2.29	Relativistic Particles
0.99c	7.09	
0.999c	23	
0.9999c	70	

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$



Postulates of STR

i) c is invariant in all inertial frame

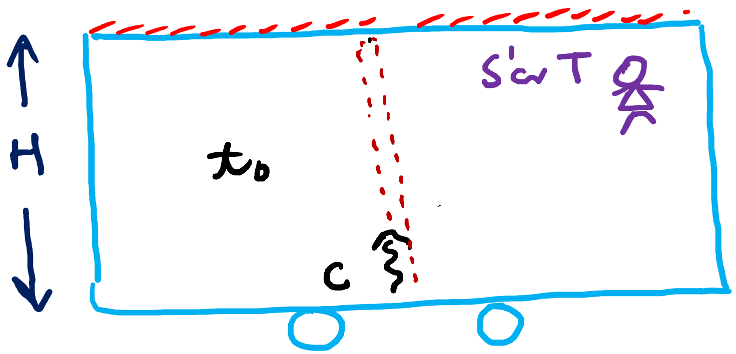
ii) All inertial frames are eq^t.

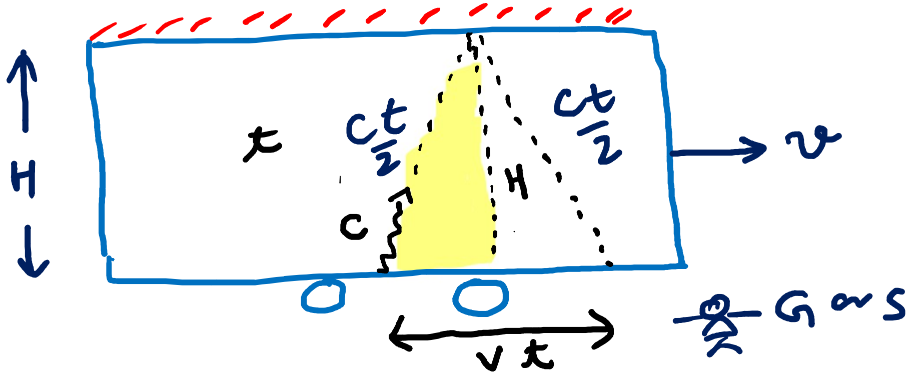
2) Time dilation :

"Moving clock runs slow"

(clock ↓ Time)

Explaination :





As viewed by T,

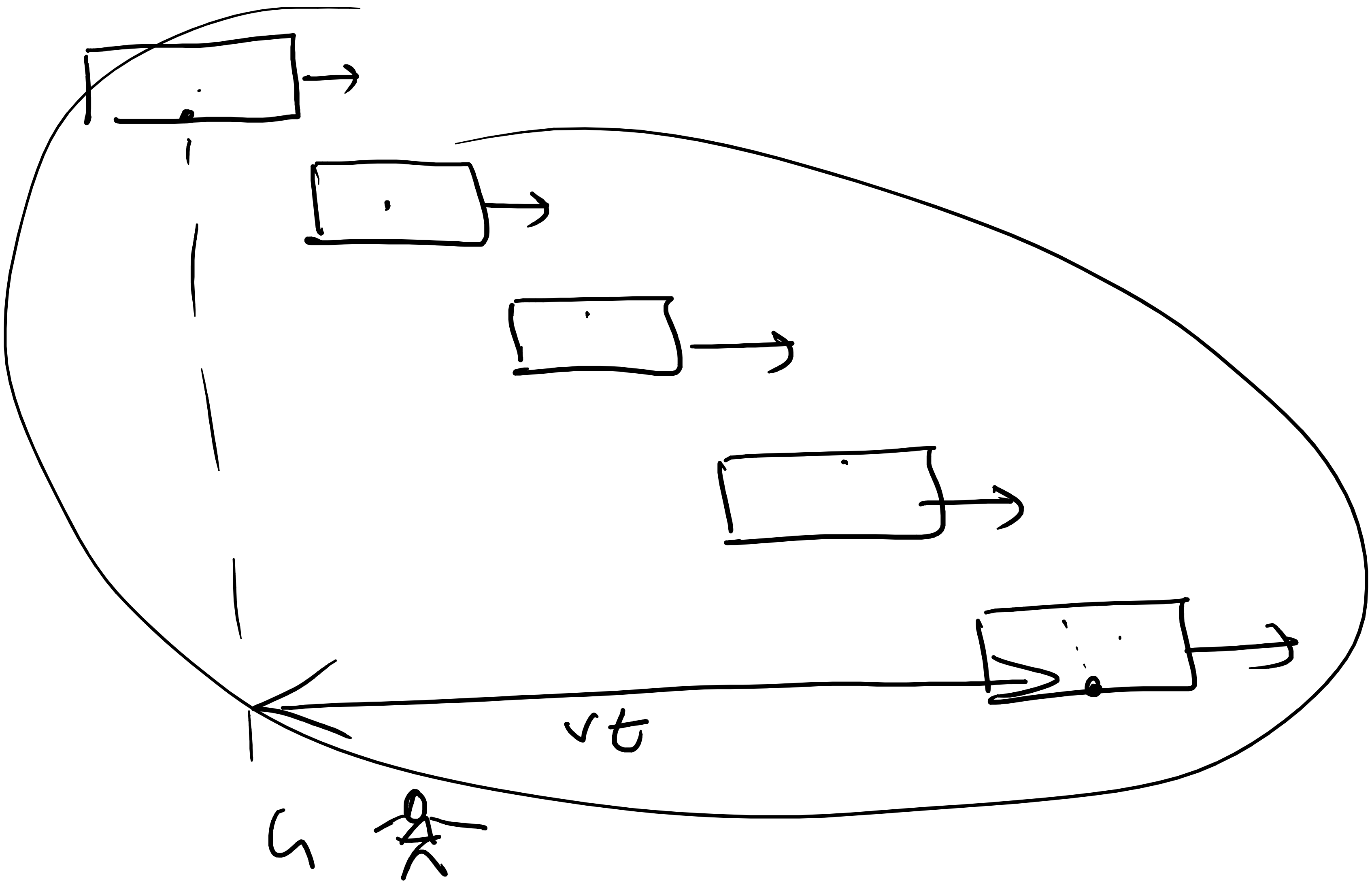
$$t_0 = \frac{H}{c} + \frac{H}{c} = \frac{2H}{c} \quad \text{— (1)}$$

As viewed by G,

$$H^2 + \left(\frac{vt}{2}\right)^2 = \left(\frac{ct}{2}\right)^2$$

or $$4H^2 = t^2(c^2 - v^2)$$

or $$2H = ct\left(1 - \frac{v^2}{c^2}\right)^{1/2}$$


vt

$$t = \frac{2H/c}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\boxed{t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}} \quad (\because \text{ eq}^n \text{ ①})$$

or

$$\boxed{t = \gamma t_0}$$

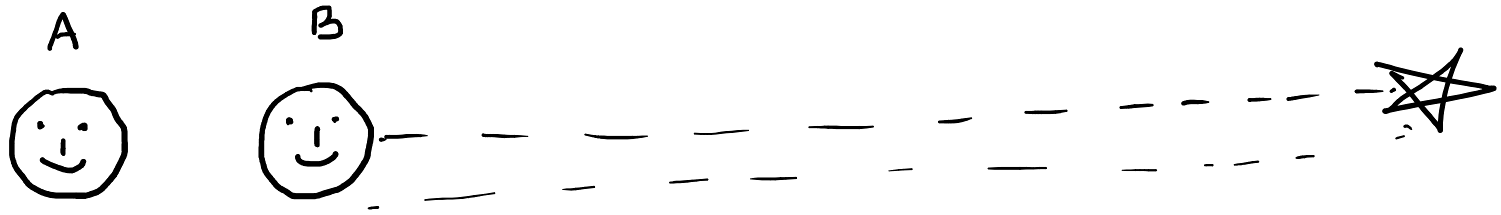
where

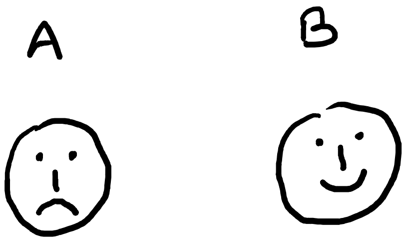
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} > 1 \Rightarrow \boxed{t > t_0}$$

(Lorentz Factor)

i.e. time gets dilated.

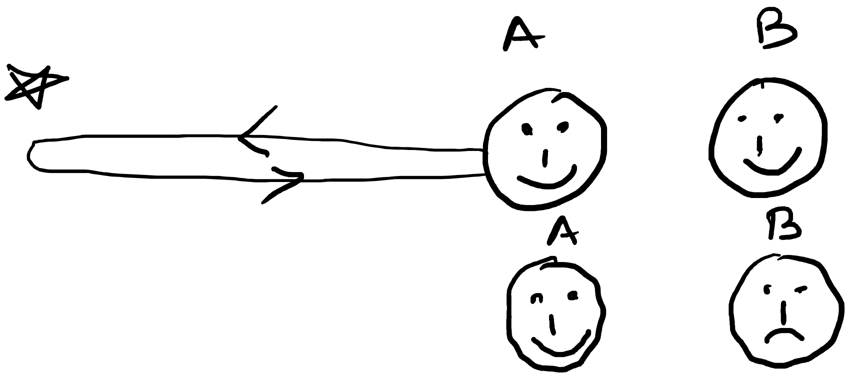
wrt to A,





"B is younger to me"

wrt B,



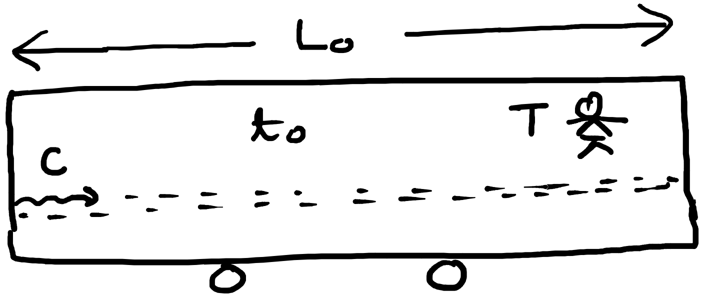
"A is younger to me"

"Twins Paradox"

3) Length contraction : "Moving rod looks shorter"

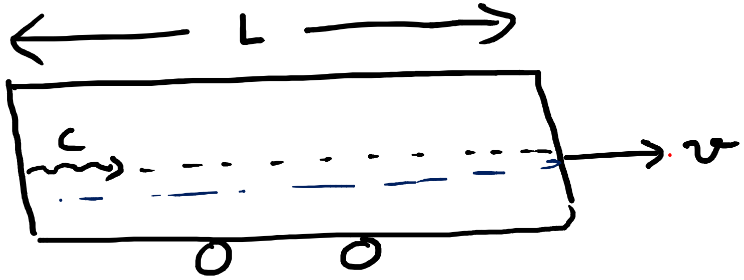
(rod ↓ space)

As viewed by T,



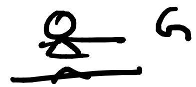
$$t_0 = \frac{2L_0}{c} \quad \text{--- (1)}$$

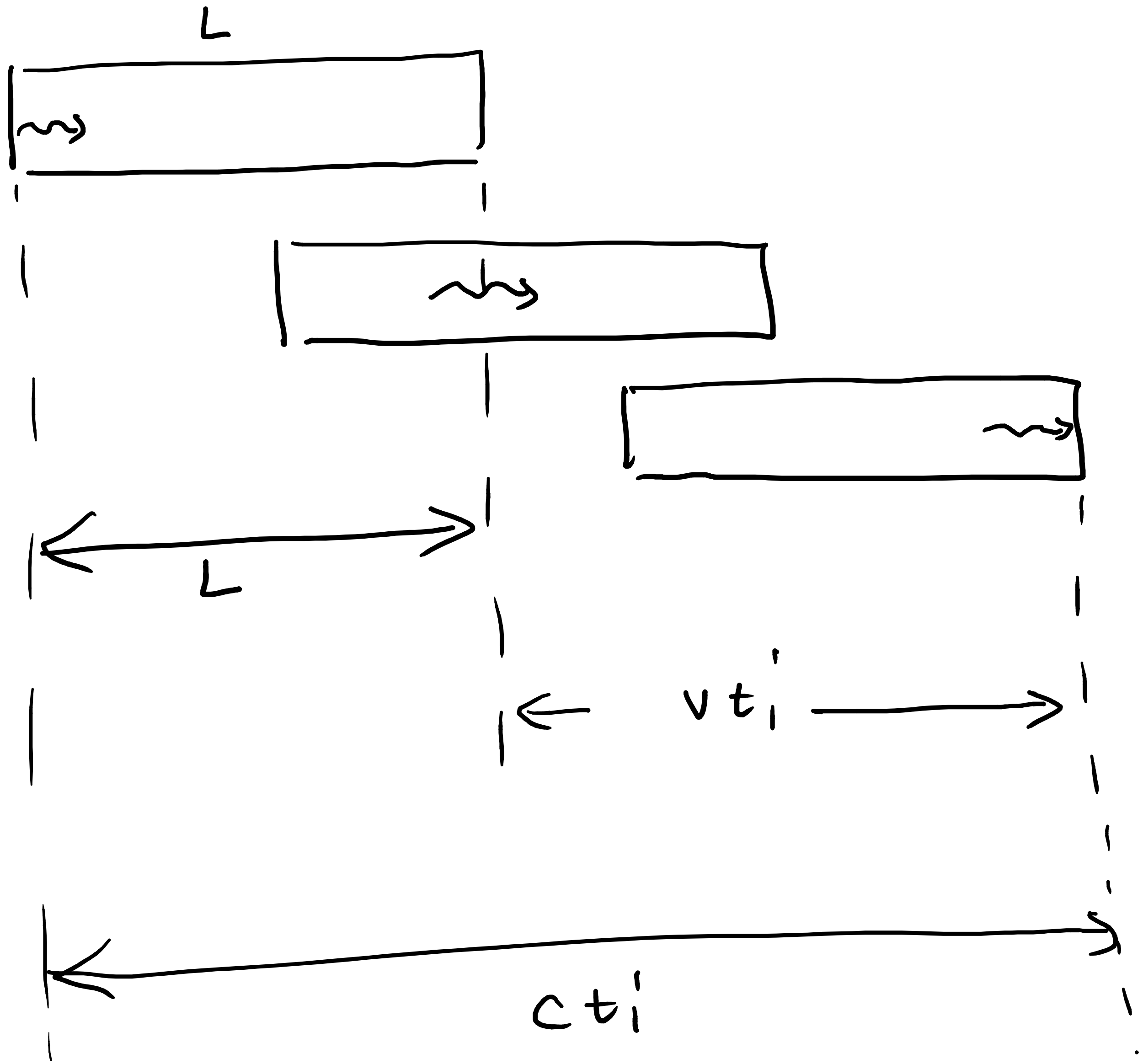
As viewed by G,



$$t = \frac{L}{c-v} + \frac{L}{c+v}$$

$$t = \frac{2Lc}{c^2 - v^2} = \frac{(2L/c)}{1 - \frac{v^2}{c^2}} \quad \text{--- (2)}$$





$$L = c t_1' - v t_1'$$

$$t_1' = \frac{L}{c-v}$$

Since we know that

$$t = \gamma t_0$$

From eq^n (1) & (2),

$$\frac{\frac{2L}{c}}{1-\frac{v^2}{c^2}} = \gamma \cdot \frac{2L_0}{c}$$

or

$$\frac{2L}{c}\gamma^2 = \gamma \frac{2L_0}{c}$$

$$L = \frac{L_0}{\gamma}$$

Since $\gamma > 1$

$$L < L_0$$

length gets contracted.

* at $v = c$,

$L = 0$, $t = \infty$

This is not possible.

LHC

$${}_0n^1 \rightarrow {}_1H^1 + {}_{-1}\beta^0 + \bar{\nu}$$

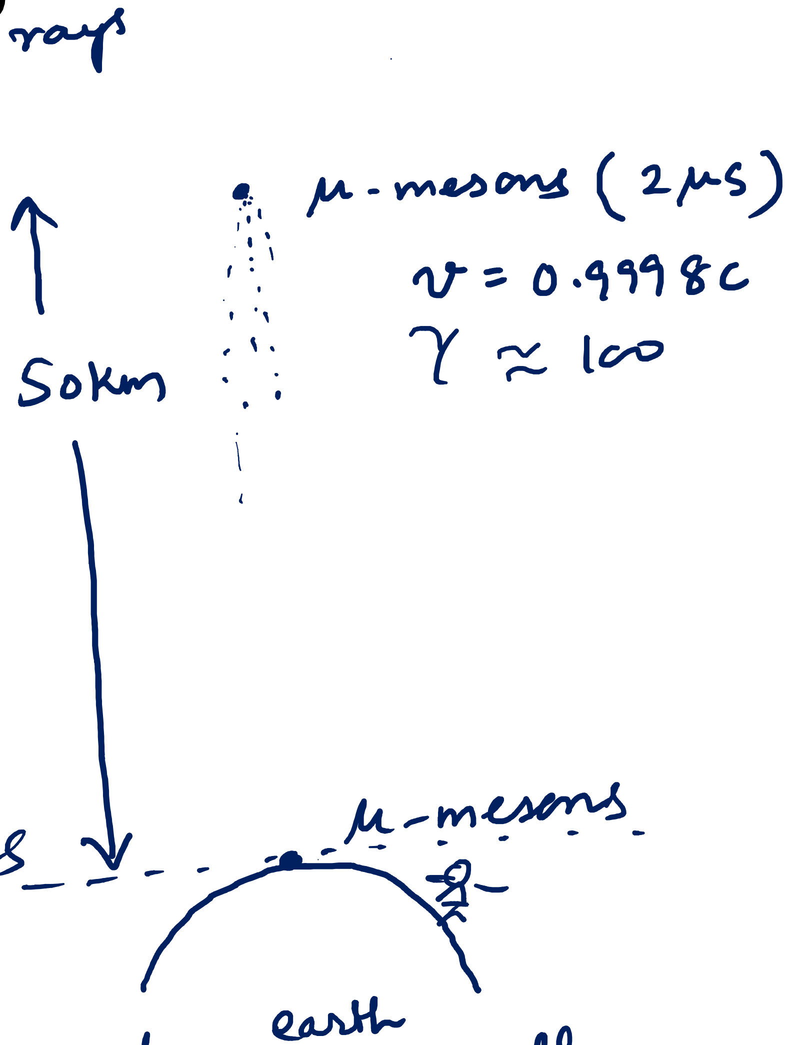
Elementary particle

- Fermi - Dirac
  - Fermions
    - e^-, p, n, e^+, ν, $\bar{\nu}$
- Bose - Einstein
  - Bosons
    - meson
      - π^0, π^-, π^+
    - mass less Boson
      - $h\nu$, ν, $\bar{\nu}$

Pauli

Yukawa

Evidence of STR or Time dilation and length contraction:-



without STR we can not explain why μ-mesons ($v \approx 0.9998c$, $t_0 = 2\mu s$) reach at the surface of earth?

<u>wrt earth</u>:- (Time dilation)

$$t = \gamma t_0 = 100 \times 2\mu s$$
$$= 200\ \mu s$$

$$\text{distance travelled} \approx 3 \times 10^8 \times 200\ \mu s$$
$$\approx 60\ \text{km}$$

i.e. μ-meson reached at surface of earth.

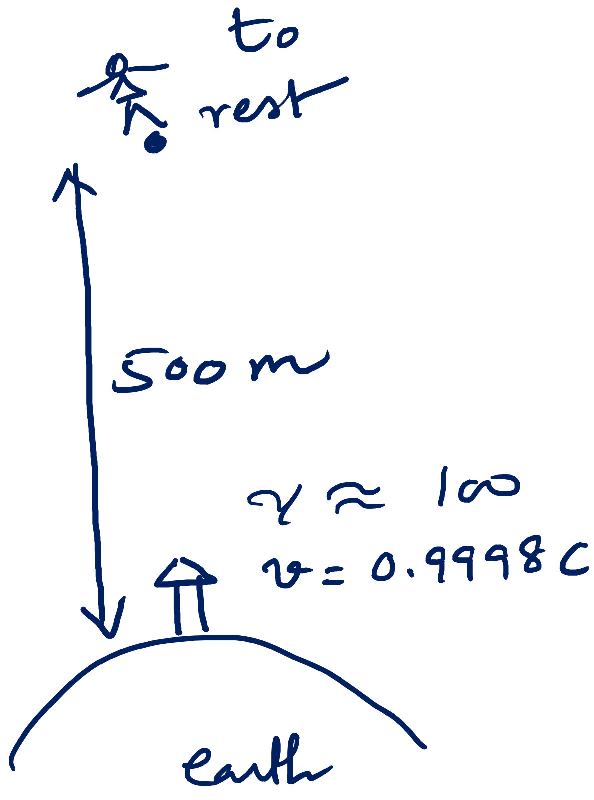
wrt μ-mesons :- (Length contraction)

distance get contracted

$$L = \frac{L_0}{\gamma} = \frac{50}{100}\,km$$

$$L \approx 500m$$

μ- meson reach at the surface of earth.



1. The average lifetime of a pi meson in its own frame of reference is 26.0 ns. (a) If the pi meson moves with speed 0.95c with respect to the Earth, what is its lifetime as measured by an observer at rest on Earth? (b) What is the average distance it travels before decaying as measured by an observer at rest on Earth?

Sol^n:

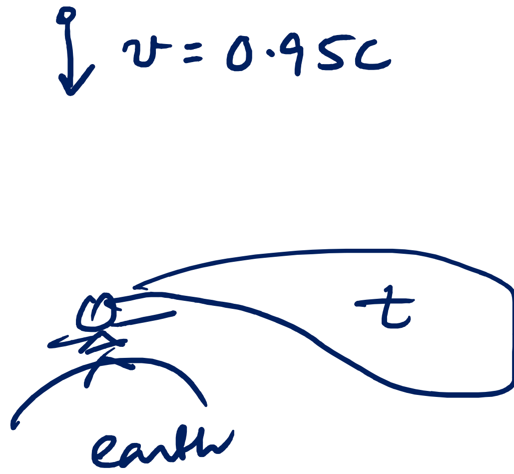
$$t_0 = 26 \text{ ns}$$

a) $v = 0.95c$, $\gamma = \frac{1}{\sqrt{1-(0.95)^2}} = 3.2$

$$t = \gamma t_0 = 3.2 \times 26 \text{ ns}$$

$$= 83.2 \text{ ns}$$

(dilated time)



b) $d = vt = 0.95c \times 83.2 \text{ ns}$

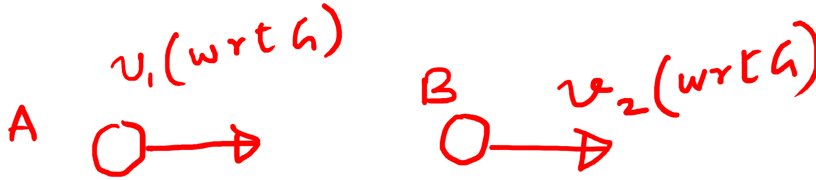
$$= 0.95 \times 3 \times 10^8 \times 83.2 \times 10^{-9}$$

$$= 2371.2 \text{ m} \approx 2.4 \text{ km}$$

Thank You

Velocity - addition theorem :-

Motivation :- If the velocity of A and B are v_1 and v_2 wrt ground. then, what is the velocity of B wrt A ?



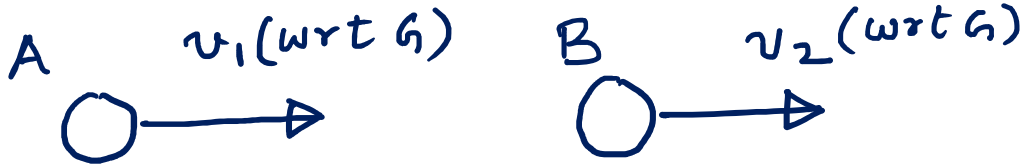
Answer :- $|v_2 - v_1|$

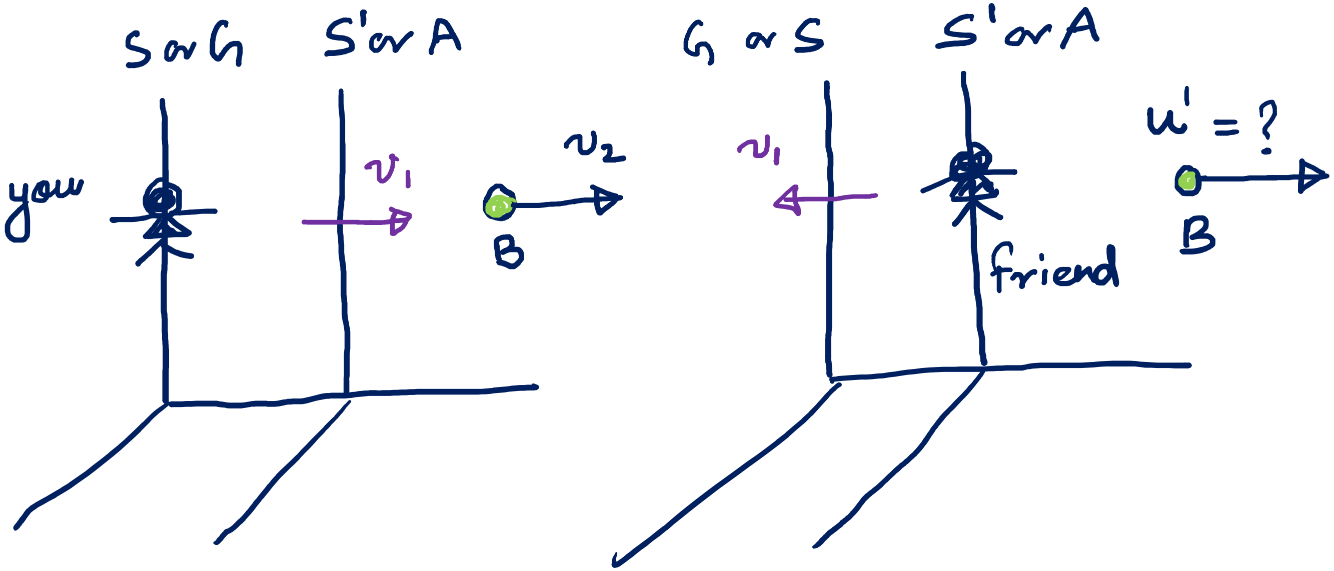
This is **incorrect** answer.

"Lets start"

Velocity-addition theorem:-

Find velocity of B wrt A.





L.T. eqns. are,

$$\Delta x' = \gamma(\Delta x - v_1 \Delta t) \quad \text{--- (1)}$$

$$\Delta t' = \gamma\left(\Delta t - \frac{v_1 \Delta x}{c^2}\right) \quad \text{--- (2)}$$

where $\gamma = \dfrac{1}{\sqrt{1 - \frac{v_1^2}{c^2}}}$

eq^n ① ÷ eq^n ② ,

$$\left(\frac{\Delta x'}{\Delta t'}\right) = \frac{\Delta x - v_1 \Delta t}{\Delta t - \frac{v_1 \Delta x}{c^2}} = \frac{\left(\frac{\Delta x}{\Delta t}\right) - v_1}{1 - \frac{v_1}{c^2}\left(\frac{\Delta x}{\Delta t}\right)}$$

velocity of ball B wrt G

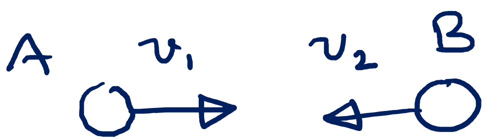
velocity of ball B wrt A

$$u' = \frac{|v_2 - v_1|}{1 - \frac{v_1 v_2}{c^2}}$$

Correction

Similarly,

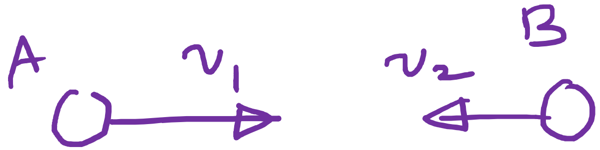
$$u' = \left|\frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}\right|$$



Key idea :-

$\Rightarrow$ $$u' = \frac{v_1 \sim v_2}{1 - \frac{v_1 v_2}{c^2}}$$

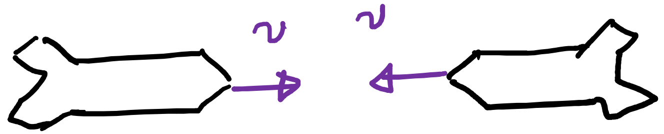
$\Rightarrow$ $$u' = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}$$



Two spaceships approach each other, each moving with the same speed as measured by a stationary observer on the Earth. Their relative speed is 0.70c. Determine the velocities of each spaceship as measured by the stationary observer on Earth.

Soln:-

$u' = 0.7c \quad , \; v = ?$



$$u' = \frac{v + v}{1 + \frac{vv}{c^2}}$$

$$0.7c = \frac{2v}{1 + \frac{v^2}{c^2}} \Rightarrow \boxed{v = \quad}$$

In units of c,

$$0.7 = \frac{2v}{1+v^2}$$

$$7 + 7v^2 = 20v$$

$$7v^2 - 20v + 7 = 0$$

$$v = \frac{20 \pm \sqrt{400 - 196}}{14}$$

$$v = \frac{20 \pm \sqrt{204}}{14} = \frac{10 \pm \sqrt{51}}{7} c$$

$\frac{10 + \sqrt{51}}{7} c$ is not possible. So answer should by $c(10 - \sqrt{51})/7$

Mass - energy equivalence :-

$$E = mc^2$$

* Mass and energy are conserved separately. before 1905

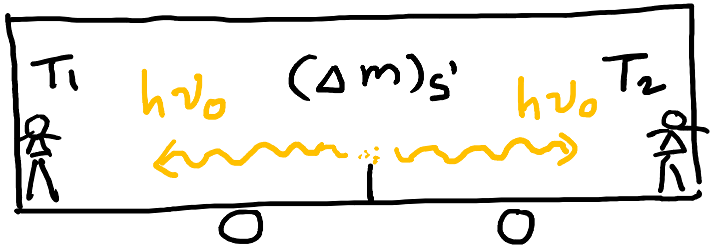
However

* (Mass + energy) is conserved. After 1905

Explaination :-

Energy released in S' frame

$$(\Delta E)_{S'} = 2h\nu_0 \quad \text{— (1)}$$



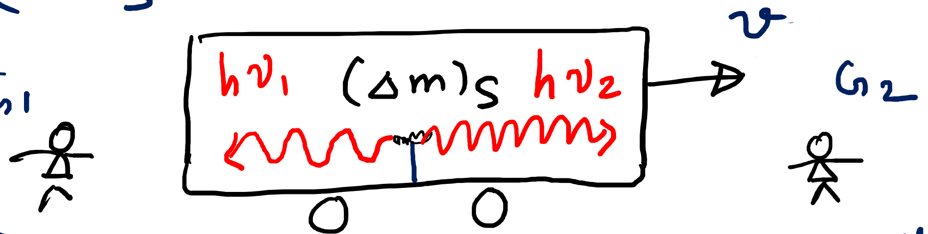
Energy released in S frame

$$(\Delta E)_S = h\nu_1 + h\nu_2 = 2h\nu_0\gamma \quad \text{— (2)}$$

"Energy is relative" $\quad (\Delta E)_S = (\Delta E)_{S'}\gamma$

Momentum conservation law in S' frame

$$0 = 0 \quad \text{— (3)}$$



$$\nu_2 = \nu_0\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}\,, \quad \nu_1 = \nu_0\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

12th std

Momentum conservation law in S frame,

$$(\Delta m)_S\, v = \frac{h\nu_2}{c} - \frac{h\nu_1}{c}$$

$$= \frac{h\nu_0}{c}\left[\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} - \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}\right]$$

$$(\Delta m)_S\, \cancel{v} = \frac{h\nu_0}{c}\gamma\,\frac{2\cancel{v}}{c} = \frac{(\Delta E)_S}{c^2}$$

$E = mc^2$

$$\boxed{(\Delta E)_S = (\Delta m)_S\, c^2} \quad \text{——— (3)}$$

Similarly,

$$(\Delta E)_{S'} = (\Delta m)_{S'}\, c^2 \quad \text{——— (4)}$$

eq^n (3) $\div$ eq^n (4)

$$\frac{(\Delta E)_s}{(\Delta E)_{s'}} = \frac{(\Delta m)_s}{(\Delta m)_{s'}}$$

(eq^n (1) and (2))

$$\Rightarrow \quad (\Delta m)_s = \gamma \, (\Delta m)_{s'}$$

$(\Delta m)_s$ → moving mass

$(\Delta m)_{s'}$ → rest mass

$$\boxed{m = \gamma \, m_0} \quad \text{or} \quad \boxed{m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

mass is relative ←

Since $\gamma > 1 \Rightarrow \boxed{m > m_0}$

Key points:-

* $m = \gamma m_0$ (moving mass)
* $E = mc^2 = \gamma m_0 c^2$ (Total energy)
* $E_0 = m_0 c^2$ (if particle is at rest)

Rest mass energy.

eg:- rest mass energy of e^- = 0.51 MeV

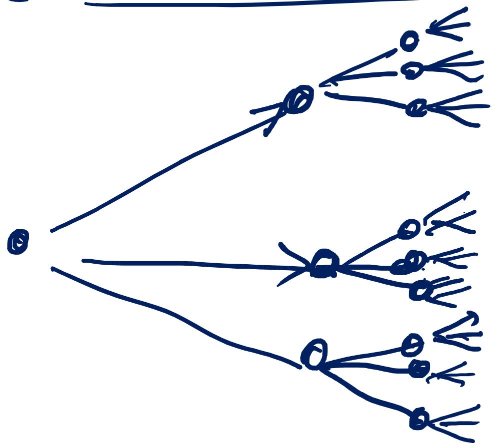
rest mass of e^- = 0.51 MeV/c^2

$$1 \text{ amu} = 931.5 \, MeV/c^2$$

1.67×10^{-27} kg.

$$e^+ + e^- = h\nu + h\nu$$

$$ {}_{92}U^{235} + {}_{0}n^{1} \rightarrow Ba + Kr + 3\,{}_{0}n^{1} + 180\,MeV $$



absorbs.

$$ {}_{92}U^{235} + {}_{92}U^{238} $$

0.3% 99.7%

Thank You

Momentum :-

$$p = mv = \gamma m_o v \quad \text{where } \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Kinetic energy :-

$$E_K = mc^2 - m_o c^2 = \gamma m_o c^2 - m_o c^2$$

$$E_K = (\gamma - 1) m_o c^2 \quad \text{(True)}$$

OR

$$E_K = \left[\left(1 - \frac{v^2}{c^2}\right)^{-1/2} - 1\right] m_o c^2$$

OR

$$E_K = \left[1 + \frac{v^2}{2c^2} + \frac{3}{8}\frac{v^4}{c^4} + \cdots - 1\right] m_o c^2$$

$$E_K = \frac{1}{2} m_o v^2 + \frac{3}{8} m_o \frac{v^4}{c^2} + \cdots$$

$$(1-x)^{-n} = 1 + nx + \frac{n(n+1)}{2!}x^2 + \cdots$$

If $v <<< c$,

$$\boxed{E_K \approx \frac{1}{2} m_0 v^2}$$ (for non-relativistic particle)

Relation b/w energy & momentum :-

Since we have,

$$E = \gamma m_0 c^2 \Rightarrow E^2 = \gamma^2 m_0^2 c^4 \quad \text{—(1)}$$

$$p = \gamma m_0 v \Rightarrow p^2 c^2 = \gamma^2 m_0^2 v^2 c^2 \quad \text{—(2)}$$

eq^n ① − eq^n ②

$$E^2 - p^2c^2 = \gamma^2 m_0^2 c^2 (c^2 - v^2)$$

$$= \gamma^2 m_0^2 c^4 \left(1 - \frac{v^2}{c^2}\right)$$

$$= \gamma^2 m_0^2 c^4 \frac{1}{\gamma^2}$$

$$= m_0^2 c^4$$

So

$$\boxed{E^2 = p^2c^2 + m_0^2 c^4}$$

Relation b/w K.E. and momentum :-

Since we have, $E_K = (\gamma - 1) m_0 c^2$ or $E_K = E - m_0 c^2$

$$p = \gamma m_0 v$$

also, $$E^2 = p^2 c^2 + m_0^2 c^4$$

$$(E_K + m_0 c^2)^2 = p^2 c^2 + m_0^2 c^4$$

$$E_K^2 + \cancel{m_0^2 c^4} + 2 E_K m_0 c^2 = p^2 c^2 + \cancel{m_0^2 c^4}$$

$$E_K (E_K + 2 m_0 c^2) = p^2 c^2$$

So $$\boxed{p = \frac{\sqrt{E_K (E_K + 2 m_0 c^2)}}{c}}$$

Mass-less particles :-

Rest mass of massless particles is zero.

$$\boxed{m_0 = 0}$$

Example :- photon, ν, $\bar{\nu}$ etc.

Since we know that

$$E^2 = p^2c^2 + m_0^2c^4$$

For massless particles,

$$E^2 = p^2c^2 \Rightarrow E = pc$$

$$\boxed{p = \frac{E}{c}}$$

$$p = \frac{E}{c}$$

For photon,

$$E = h\nu = \frac{hc}{\lambda}$$

So

$$\boxed{p = \frac{h}{\lambda}} \quad \text{or} \quad \boxed{\lambda = \frac{h}{p}}$$

-: Vector calculus :- (Maths II)

Del operator :-

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

∇ → vector

$\nabla \cdot \vec{A}$ — scalar ($\vec{A}$ → vector)
or
div $\vec{A}$

$\nabla \times \vec{A}$ — vector ($\vec{A}$ → vector)
or
Curl $\vec{A}$

$\nabla \phi$ — vector (ϕ → scalar)
or
grad ϕ

Divergence of a vector :- (How much field is spreading)

$$\text{div}\,\vec{A} = \nabla \cdot \vec{A} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \cdot \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)$$

$$\boxed{\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}}$$

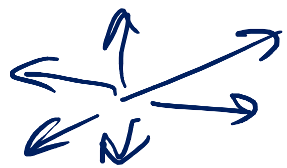
eg:- if $\vec{A} = 2xy\,\hat{i} + yz^2\,\hat{k}$

$$\text{div}\,\vec{A} = 2y + 2yz = 2y(1+z)$$

Ans

if $\vec{A} = x\hat{i} + y\hat{j} + z\hat{k}$

$\text{div}\,\vec{A} = 3$



Curl of a vector :- (How much field is rotating)

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

eg:

$$\vec{A} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = 0$$

Key points :-

* If $\operatorname{div} \vec{A} = 0$ then we can say that field vector $\vec{A}$ is solenoidal.

* If $\operatorname{curl} \vec{A} = 0$ then " " " " " " $\vec{A}$ is conservative and irrotational.

Note:-

$$\lambda = \frac{dq}{dl} \quad , \quad \sigma = \frac{dq}{dS} \quad , \quad \rho = \frac{dq}{dZ}$$

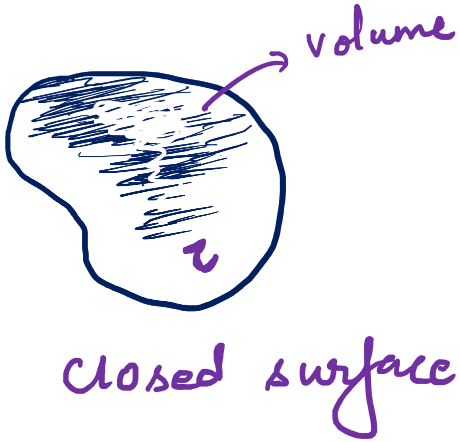
λ ↓ line charge density

σ Surface charge density.

ρ volume charge density

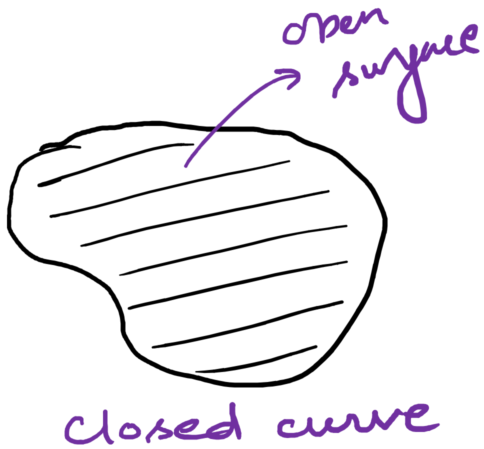
Gauss div theorem :-

$$\oint_S \vec{F} \cdot d\vec{S} = \int_\tau \operatorname{div} \vec{F} \, d\tau$$



Stock's theorem:-

$$\oint_C \vec{F} \cdot d\vec{l} = \int_S \operatorname{Curl} \vec{F} \cdot d\vec{S}$$



Unit 2 :- EMT

Gauss's law :-

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{Q_{in}}{\epsilon}$$ → Integral form

or

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{1}{\epsilon} \int_\tau \rho \, d\tau$$ (→ vol. element) $$\left(\because \rho = \frac{dq}{d\tau} \right)$$

or

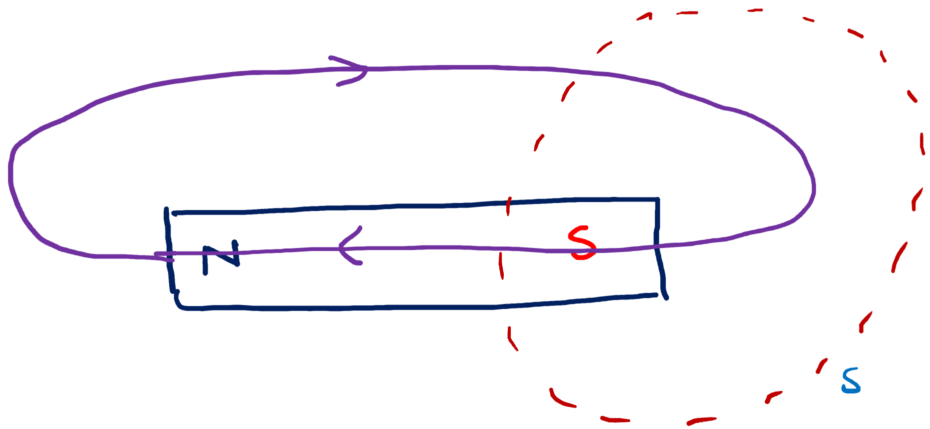
$$\oint_S \vec{E} \cdot d\vec{S} = \int_\tau \left(\frac{\rho}{\epsilon} \right) d\tau$$

Using Gauss div. theorem

$$\int_\tau \text{div} \vec{E} \, d\tau = \int_\tau \frac{\rho}{\epsilon} \, d\tau \Rightarrow \boxed{\text{div} \vec{E} = \frac{\rho}{\epsilon}}$$ → diff form

Thank You

Gauss's Law in magetic field :



Since magnetic field lines are closed loop so

$$\oint_S \vec{B} \cdot d\vec{S} = 0$$

This is integral form of Gauss's Law in magnetic field.

Using Gauss. div theorem,

M. Imp

$$\int_\tau \mathrm{div}\vec{B}\, d\tau = 0 \Rightarrow \boxed{\mathrm{div}\,\vec{B} = 0 \quad \text{OR} \quad \nabla \cdot \vec{B} = 0}$$

* $\nabla \cdot \vec{B} = 0$ says that

$\Rightarrow$ Magnetic Monopole does not exist.

OR

$\Rightarrow$ Magnetic field lines are closed loop.

Faraday law :-

induced emf. (Unit of emf volt)

$$\mathcal{E} = -\frac{d\phi}{dt}$$

or

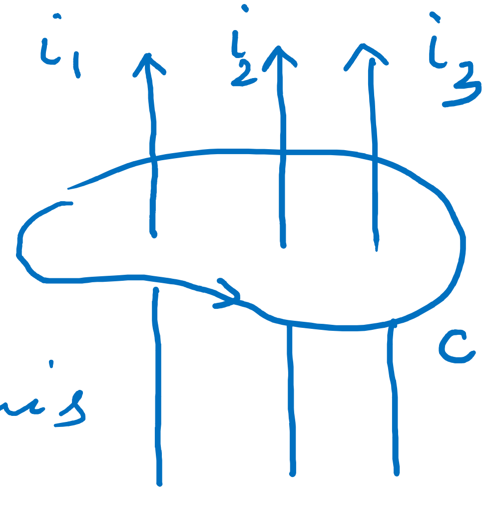
$$\oint_C \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{S} \qquad \left(\because \mathcal{E} = \oint_C \vec{E}\cdot d\vec{l}\right)$$

this is integral form of Faraday law.

Using stok's theorem,

$$\int_S \text{Curl}\,\vec{E}\cdot d\vec{S} = -\int_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S} \Rightarrow \boxed{\text{Curl}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

Ampere's law :-

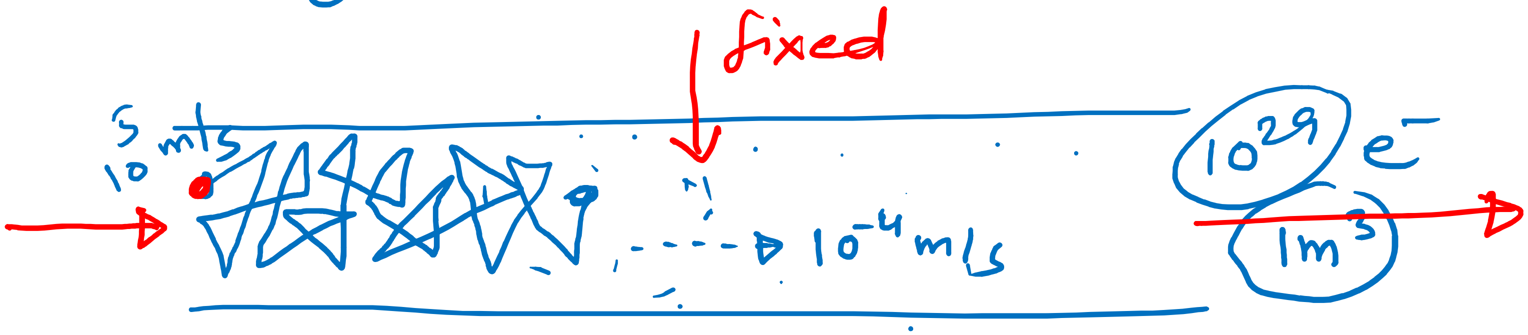


$$\oint_c \vec{B} \cdot d\vec{l} = \mu I_{in}$$

This is integral form of Ampere's law.

Now,

$$\oint_c \vec{B} \cdot d\vec{l} = \mu \int \vec{J} \cdot d\vec{s} \quad \left(\because J = \frac{dI}{ds} \right)$$



Using stock's theorem,

$$\int_S \text{Curl}\,\vec{B}\cdot d\vec{S} = \int_S \mu\vec{J}\cdot d\vec{S}$$

OR

$$\boxed{\text{Curl}\,\vec{B} = \mu\vec{J}}$$

This is diff. form of Ampere's law.

This is EMT before Maxwell.

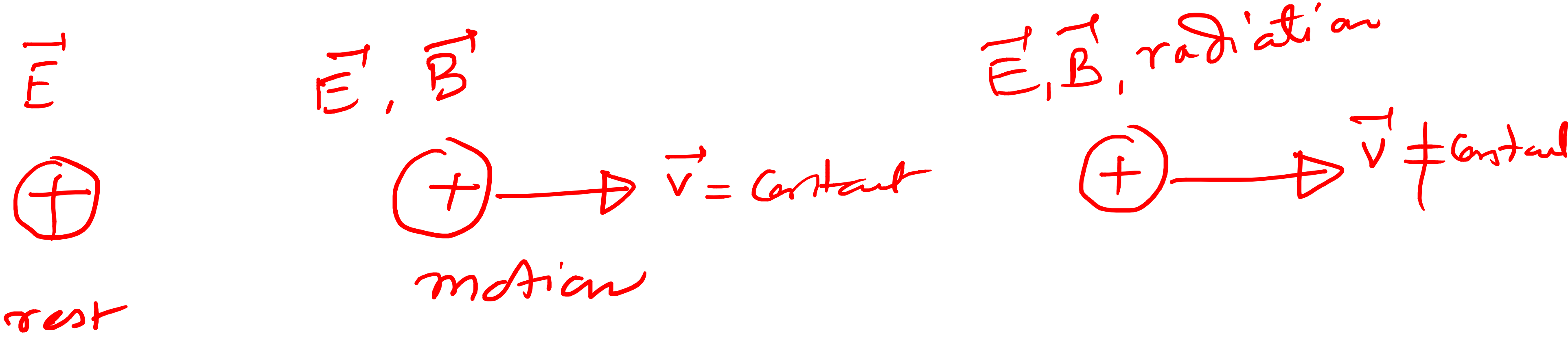
*

i) $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon}$

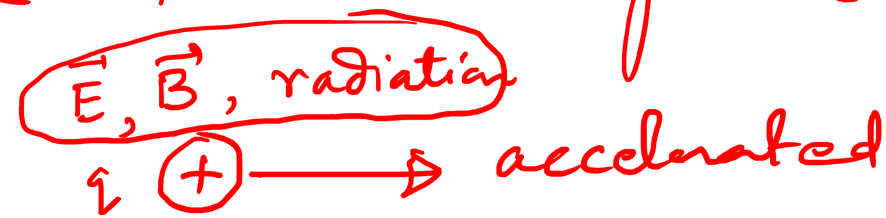
ii) $\nabla \cdot \vec{B} = 0$

iii) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

iv) $\nabla \times \vec{B} = \mu \vec{J}$



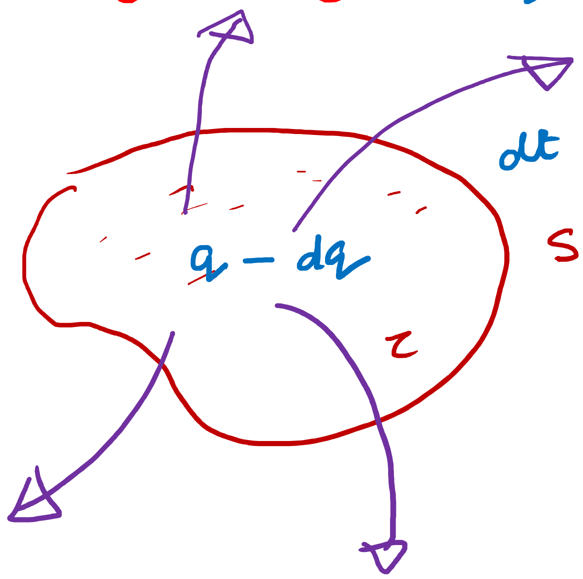
Note:- Electro-magnetic field is non-conservative in nature. It is dissipative.



Thank You

Equation of continuity: (based on conservation of charge)

$$i = -\frac{dq}{dt} \qquad \text{—— (1)}$$



Here −ve sign indicates that charge enclosed in S decay with time.

Since we know that,

$$i = \oint_S \vec{J} \cdot d\vec{s} \qquad \text{—— (2)}$$

and

$$q = \int_\tau \rho \, d\tau \qquad \text{—— (3)}$$

Put i and q in eq^n ①,

$$\oint_S \vec{J}\cdot d\vec{s} = -\frac{d}{dt}\int_\tau \rho\, d\tau$$

$$\oint_S \vec{J}\cdot d\vec{s} = -\int_\tau \frac{\partial \rho}{\partial t}\, d\tau$$

Using Gauss-div theorem

$$\int_\tau \operatorname{div}\vec{J}\, d\tau = -\int \frac{\partial \rho}{\partial t}\, d\tau \Rightarrow \boxed{\operatorname{div}\vec{J} + \frac{\partial \rho}{\partial t} = 0}$$

$$\boxed{\nabla\cdot\vec{J} + \frac{\partial \rho}{\partial t} = 0}$$

based on conservation of charge.

This is called as eq^n of continuity.

What is the problem with Ampere's Law??

Since we have Ampere's law,

$$\text{curl } \vec{B} = \mu \vec{J} \qquad \text{(diff. form)}$$

or

$$\nabla \times \vec{B} = \mu \vec{J}$$

Taking div both side.

$$\nabla \cdot (\nabla \times \vec{B}) = \mu (\nabla \cdot \vec{J})$$

Since $\nabla \cdot (\nabla \times \vec{B}) = 0$. So

$$\boxed{\nabla \cdot \vec{J} = 0}$$

This voilate conservation of charge.

i.e. Ampere's law i.e. Curl $\vec{B} = \mu \vec{J}$ needs corrections.

Modified Ampere's law :-

since eqⁿ of continuity is

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0 \qquad \text{—— (1)}$$

we know that Gauss's law,

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon} \Rightarrow \rho = \epsilon \nabla \cdot \vec{E}$$

put ρ in eqⁿ (1)

$$\nabla \cdot \vec{J} + \frac{\partial}{\partial t}(\epsilon \nabla \cdot \vec{E}) = 0$$

OR

$$\nabla \cdot \left(\vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t}\right) = 0$$

i.e Maxwell's suggested that,

$$\vec{J} \longrightarrow \vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t} \quad \text{in}$$

Ampere's law.

i.e. modified Ampere's law becomes

$$\nabla \times \vec{B} = \mu \left(\vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t} \right)$$

Note :-

$$\nabla \times \vec{B} = \mu \vec{J} + \mu \epsilon \frac{\partial \vec{E}}{\partial t}$$ ✓

Taking div both side

$$0 = \mu \nabla \cdot \vec{J} + \mu \epsilon \frac{\partial}{\partial t}(\nabla \cdot \vec{E})$$

$$0 = \mu \nabla \cdot \vec{J} + \mu \epsilon \frac{\partial}{\partial t}\left(\frac{\rho}{\epsilon}\right)$$

$$\boxed{\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0}$$

Maxwell's eq^{n}s :-

i) $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon}$ (Gauss law)

ii) $\nabla \cdot \vec{B} = 0$ (mono pole does not exist)

iii) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (Faraday law)

iv) $\nabla \times \vec{B} = \mu \vec{J} + \mu \epsilon \frac{\partial \vec{E}}{\partial t}$ (Modi. Amp. law)

Maxwell's eq^n in free space :-

$$\rho = 0 \quad , \quad \vec{J} = 0 \quad , \mu \rightarrow \mu_0 \quad , \epsilon \rightarrow \epsilon_0$$

i) $\nabla \cdot \vec{E} = 0$

ii) $\nabla \cdot \vec{B} = 0$

iii) $\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

iv) $\nabla \times \vec{B} = \mu_0 \epsilon_0 \dfrac{\partial \vec{E}}{\partial t}$

How Maxwell's eq^n support propagation EM waves in free space :-

Taking curl both side in eq^n(iii)

$$\nabla \times (\nabla \times \vec{E}) = -\frac{\partial}{\partial t}(\nabla \times \vec{B})$$

$$\nabla(\nabla\cdot\vec{E}) - (\nabla\cdot\nabla)\vec{E} = -\frac{\partial}{\partial t}(\nabla\times\vec{B})$$

Using eqⁿ (i) and (iv)

$$0 - \nabla^2\vec{E} = -\frac{\partial}{\partial t}\left(\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}\right)$$

$$\boxed{\nabla^2\vec{E} = \mu_0\epsilon_0\frac{\partial^2\vec{E}}{\partial t^2}}$$

Again taking curl both side in eqⁿ (iv)

$$\underbrace{\nabla\times(\nabla\times\vec{B})} = \mu_0\epsilon_0\frac{\partial}{\partial t}(\nabla\times\vec{E})$$

$$\nabla(\nabla\cdot\vec{B}) - \nabla^2\vec{B} = \mu_0\epsilon_0 \frac{\partial}{\partial t}(\nabla\times\vec{E})$$

Using eq^n (ii) and (iii)

$$0 - \nabla^2\vec{B} = \mu_0\epsilon_0 \frac{\partial}{\partial t}\left(-\frac{\partial\vec{B}}{\partial t}\right)$$

$$\boxed{\nabla^2\vec{B} = \mu_0\epsilon_0 \frac{\partial^2\vec{B}}{\partial t^2}}$$

Note:-

$$\psi = a \sin(\omega t - Kx)$$

So

$$\left.\begin{aligned} \frac{\partial^2 \psi}{\partial x^2} &= K^2 \psi \\ \frac{\partial^2 \psi}{\partial t^2} &= \omega^2 \psi \end{aligned}\right\} \Rightarrow \frac{\left(\frac{\partial^2 \psi}{\partial x^2}\right)}{\left(\frac{\partial^2 \psi}{\partial t^2}\right)} = \frac{K^2}{\omega^2}$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\left(\frac{\cancel{2\pi}}{\lambda}\right)^2}{(\cancel{2\pi} n)^2} \frac{\partial^2 \psi}{\partial t^2}$$

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}}$$

In general,

$$\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$ (Laplacian operator)

Hence

$$\nabla^2(\vec{E} \text{ or } \vec{B}) = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2}(\vec{E} \text{ or } \vec{B})$$

This is the diff. eqn of progressive wave like

$$\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

On comparing

$$\frac{1}{v^2} = \mu_0 \epsilon_0 \Rightarrow v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$= 3 \times 10^8 \text{ m/s}$$

$$v = c$$

Thus,

$$\nabla^2 (\vec{E} \text{ or } \vec{B}) = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} (\vec{E} \text{ or } \vec{B})$$

Thank You

$$\nabla^2 \vec{E} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{B} = \frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\equiv \quad \boxed{\nabla^2 \psi = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}}$$

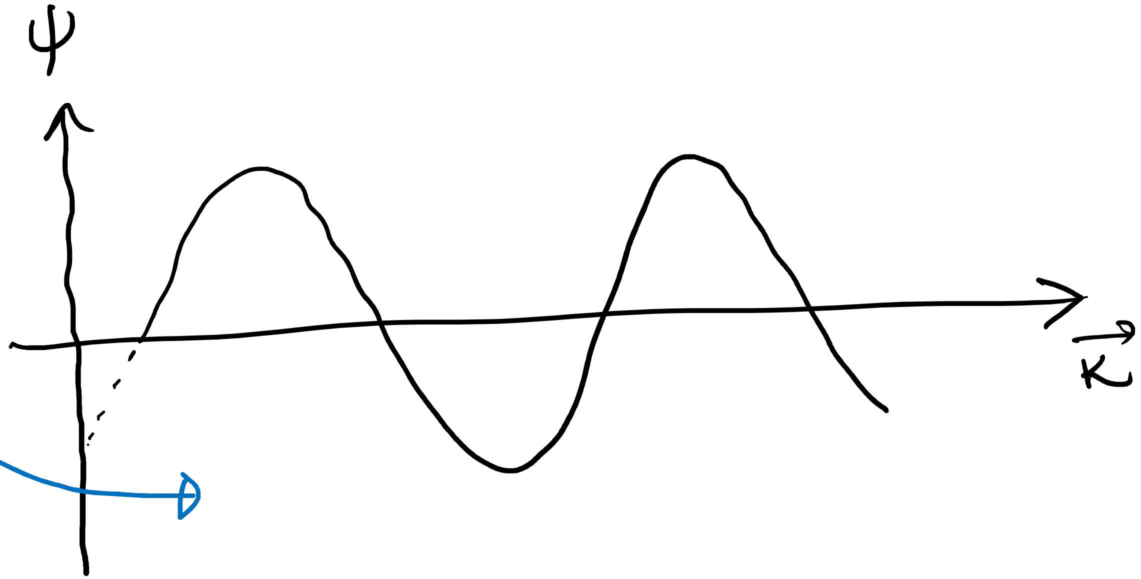
$$y = a \sin(\omega t - \vec{k}\cdot\vec{r})$$

or

$$y = a\, e^{i(\omega t - \vec{k}\cdot\vec{r})}$$

or

$$y = a \cos(\omega t - \vec{k}\cdot\vec{r})$$



$$e^{i\theta} = \cos\theta + i\sin\theta$$

Diff form of SHO :-

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

$$\Rightarrow D^2 + \omega^2 = 0 \qquad \left(\frac{d}{dt} = D\right)$$

$$\Rightarrow D = \pm i\omega$$

$$\boxed{x = A e^{i\omega t} + B e^{-i\omega t}}$$

↙ Boundary Condition

$\vec{E}$ & $\vec{B}$ satisfy diff. eqn.

$$\nabla^2 \vec{E} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}$$

&

$$\nabla^2 \vec{B} = \frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2}$$

$k_x x + k_y y + k_z z$

the soln of these diff eqn.

$$\vec{E}(x,y,z,t) = \vec{E_0}\, e^{i(\vec{k}\cdot\vec{r} - wt)}$$

$$\vec{B}(x,y,z,t) = \vec{B_0}\, e^{i(\vec{k}\cdot\vec{r} - wt)}$$

Now

$$\frac{\partial \vec{E}}{\partial t} = E_0 e^{i(\vec{k}\cdot\vec{r} - wt)} \cdot (-iw) = -iw\,\vec{E}$$

$$\frac{\partial}{\partial t} = -iw$$ (taking operator)

and

$$\frac{\partial \vec{E}}{\partial x} = i k_x \vec{E} \Rightarrow \frac{\partial}{\partial x} = i k_x \quad \times \hat{i}$$

$$\frac{\partial \vec{E}}{\partial y} = i k_y \vec{E} \Rightarrow \frac{\partial}{\partial y} = i k_y \quad \times \hat{j}$$

$$\frac{\partial \vec{E}}{\partial z} = i k_z \vec{E} \Rightarrow \frac{\partial}{\partial z} = i k_z \quad \times \hat{k}$$

On combining.

$$\Rightarrow \vec{\nabla} = i \vec{K} \quad \text{(operator)}$$

Since Maxwell's eqⁿ in Free space

$$\nabla \cdot \vec{E} = 0 \Rightarrow i\vec{K}\cdot\vec{E} = 0 \Rightarrow \vec{K}\cdot\vec{E} = 0 \Rightarrow \boxed{\vec{K} \perp \vec{E}}$$

$$\nabla \cdot \vec{B} = 0 \Rightarrow i\vec{K}\cdot\vec{B} = 0 \Rightarrow \vec{K}\cdot\vec{B} = 0 \Rightarrow \boxed{\vec{K} \perp \vec{B}}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow i\vec{K}\times\vec{E} = +i\omega\vec{B} \Rightarrow \boxed{\vec{B} = \frac{\vec{K}\times\vec{E}}{\omega}}$$

$$\Rightarrow \vec{B} \perp \vec{K} \ \& \ \vec{E}$$

$$\nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \Rightarrow i\vec{K}\times\vec{B} = \frac{1}{c^2}\cdot(-i\omega\vec{E})$$

$$\Rightarrow c\vec{K}\times\vec{B} = -\frac{\omega}{c}\vec{E}$$

$$\Rightarrow c\vec{K}\times\vec{B} = -\frac{\omega}{\omega/K}\vec{E}$$

$$= \boxed{\vec{E} = -(\hat{K}\times\vec{B})c}$$

Light waves are transverse

Extra

$$y = a\sin(2x - 4t)$$

$$y = a\sin(kx - \omega t)$$

$$v = \frac{\omega}{k} = \frac{2\pi n}{\frac{2\pi}{\lambda}} = n\lambda$$

$$k = 2 \quad , \quad \omega = 4$$

$$\frac{2\pi}{\lambda} = 2 \quad , \quad 2\pi n = 4$$

$$\lambda = \frac{2\pi}{2} \quad , \quad n = \frac{4}{2\pi}$$

$$v = n\lambda = \frac{2\pi}{2} \cdot \frac{4}{2\pi} = \frac{4}{2} = 2$$

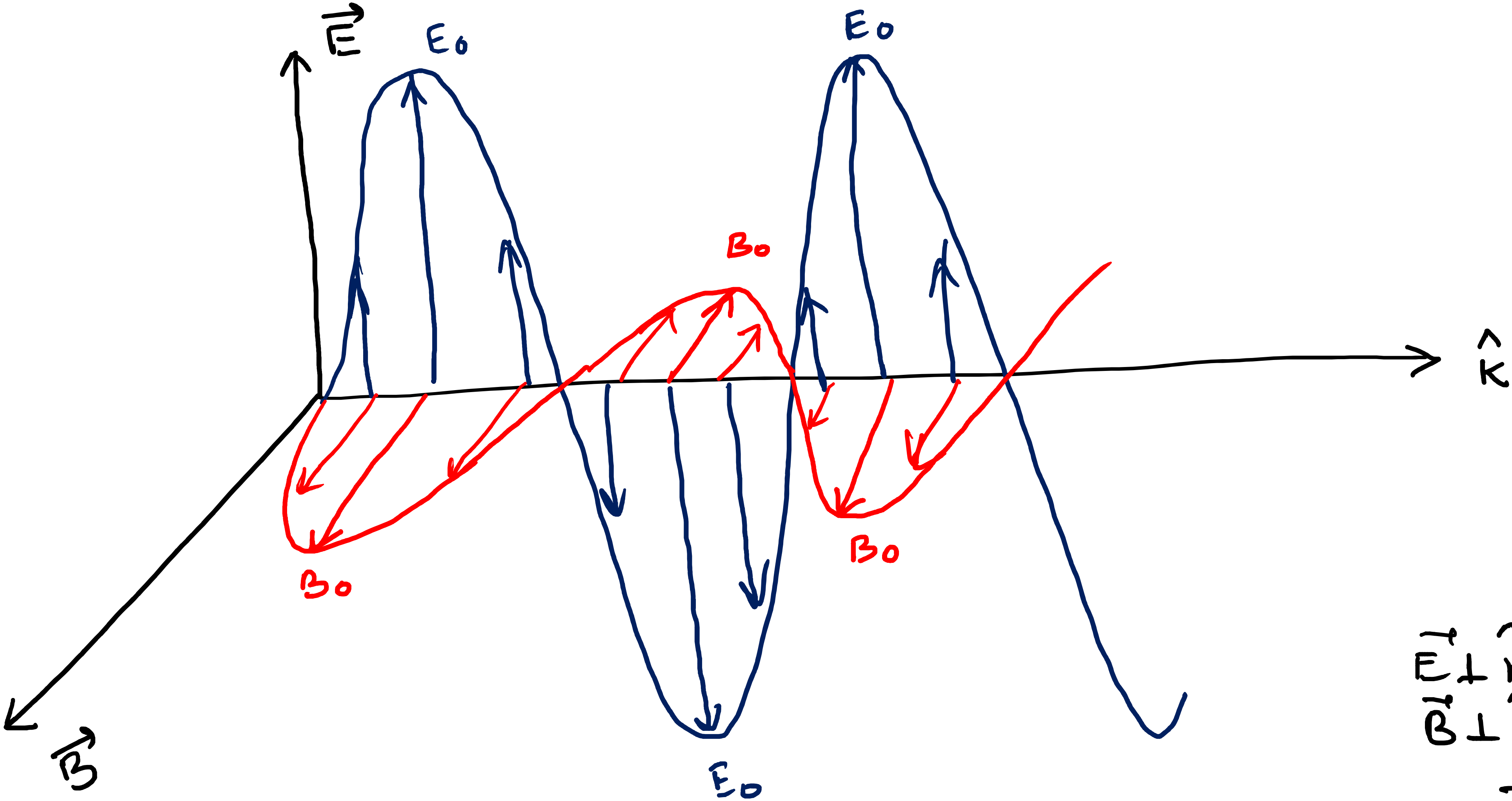
Since we have

$$\vec{B} = \frac{\vec{K} \times \vec{E}}{\omega}$$

Taking magnitude

$$B = \frac{|\vec{K}|\,|\vec{E}|\,\sin 90^\circ}{\omega} = \frac{K\,E}{\omega} = \frac{E}{c}$$

$$\boxed{\frac{|E|}{|B|} = c}$$



$\vec{E} \perp \hat{k}$ $\quad$ $\vec{E} \perp \vec{B}$

$\vec{B} \perp \hat{k}$

$$\vec{B} = \frac{\vec{k} \times \vec{E}}{\omega}$$

Maxwell's eqns in conducting media:

i) $\nabla \cdot \vec{E} = 0$ (since $\rho = 0$ in conductors)

ii) $\nabla \cdot \vec{B} = 0$

iii) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

iv) $\nabla \times \vec{B} = \mu\sigma\vec{E} + \mu\epsilon\frac{\partial \vec{E}}{\partial t}$ ($\because \vec{J} = \sigma\vec{E}$)

Taking curl of eqn (iii),

$$\nabla \times (\nabla \times \vec{E}) = -\frac{\partial}{\partial t}(\nabla \times \vec{B})$$

or $$\nabla(\nabla \cdot \vec{E}) - \nabla^2\vec{E} = -\frac{\partial}{\partial t}\left(\mu\sigma\vec{E} + \mu\epsilon\frac{\partial \vec{E}}{\partial t}\right)$$ (using eqn iv)

or $$0 - \nabla^2\vec{E} = -\mu\sigma\frac{\partial \vec{E}}{\partial t} - \mu\epsilon\frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2\vec{E} - \mu\sigma\frac{\partial\vec{E}}{\partial t} - \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2} = 0 \quad \text{—— (v)}$$

Similarly,

$$\nabla^2\vec{B} - \mu\sigma\frac{\partial\vec{B}}{\partial t} - \mu\epsilon\frac{\partial^2\vec{B}}{\partial t^2} = 0 \quad \text{—— (vi)}$$

Since we have

$$\nabla \equiv i K^* \quad \text{and} \quad \frac{\partial}{\partial t} \equiv -i\omega$$

using this in eq^n (v) & eq^n (vi) we get

$$-\vec{K}^{*2} + i\mu\sigma\omega + \mu\epsilon\omega^2 = 0 \quad \text{—— (vii)}$$

take $\vec{K}^* = \vec{\alpha} + i\vec{\beta}$ (complex number)

then $K^2 = \alpha^2 - \beta^2 + 2i\alpha\beta = \mu\epsilon\omega^2 + i\mu\sigma\omega$

Compare Real and imaginary parts.

$$\left.\begin{aligned} \alpha^2 - \beta^2 &= \mu \epsilon \omega^2 \\ 2\alpha\beta &= \mu \sigma \omega \end{aligned}\right\}$$ solve for α & β

$$\alpha^2 + \beta^2 = \sqrt{(\alpha^2-\beta^2)^2 + 4\alpha^2\beta^2} = \sqrt{\mu^2\epsilon^2\omega^4 + \mu^2\sigma^2\omega^2}$$

we get α & β,

$$\alpha = \left[\frac{1}{2}\mu\epsilon\omega^2 + \frac{1}{2}\sqrt{\mu^2\epsilon^2\omega^4 + \mu^2\sigma^2\omega^2}\right]^{1/2}$$

$$\beta = \left[\frac{1}{2}\sqrt{\mu^2\epsilon^2\omega^4 + \mu^2\sigma^2\omega^2} - \frac{1}{2}\mu\epsilon\omega^2\right]^{1/2}$$

OR

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{1/2}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]^{1/2}$$

Sol^n of eq^n (v) & (vi)

$$\vec{E} = \vec{E}_0 \, e^{i(\vec{K}^* \cdot \vec{r} - \omega t)}$$

($\vec{K}^* \to (\alpha + i\beta)$)

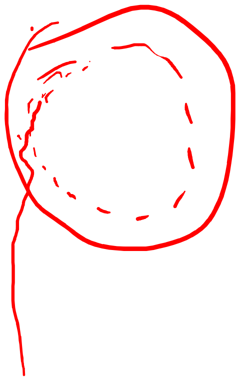
$$\vec{B} = \vec{B}_0 \, e^{i(\vec{K}^* \cdot \vec{r} - \omega t)}$$

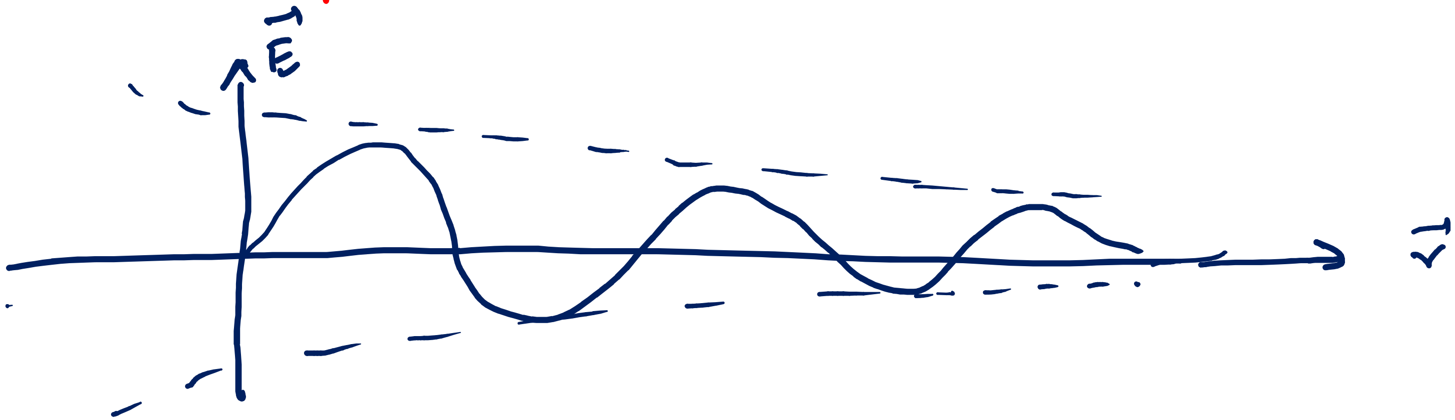
or

$$\vec{E} = \vec{E}_0 \, e^{-\vec{\beta} \cdot \vec{r}} \, e^{i(\vec{\alpha} \cdot \vec{r} - \omega t)}.$$

$e^{-\vec{\beta} \cdot \vec{r}}$ → exp. fall in amp.

$e^{i(\vec{\alpha} \cdot \vec{r} - \omega t)}$ → Oscillatory motion





In one dim.

$$\vec{E} = \vec{E}_0 e^{-\beta x} e^{i(\alpha x + \omega t)}$$

at $x = \frac{1}{\beta}$ amp. becomes $\frac{1}{e}$ times of $\vec{E}_0$

Skin depth (δ):

The distance at which electric field decreases to $\frac{1}{e}$ times the max value of amp. of electric field, is called as skin depth.

~~$\delta = \frac{\ldots}{e}$~~ or

$$\boxed{\delta = \frac{1}{\beta}}$$

or

$$\delta = \frac{1}{\omega}\sqrt{\frac{2}{\mu\epsilon}}\left[\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2} - 1\right]^{-1/2}$$

* for good conductor :-

$$\frac{\sigma}{\epsilon\omega} >> 1$$

so neglect 1 as compared to $\frac{\sigma^2}{\epsilon^2\omega^2}$.

$$\boxed{\delta \equiv \sqrt{\frac{2}{\mu\sigma\omega}}}$$

(Numerical)

Skin depth for good conductors.

γ, X, U.V., VISIBLE, IR, micro, radio

more freq. ⟶ less freq.

Unit - 1 to Unit - 5

all units.

POYNTING'S THEOREM

(Work- energy theorem)

- In electrodynamics Poynting's theorem is a statement of conservation of energy for the electromagnetic field.
- Poynting's theorem is analogous to the work-energy theorem in classical mechanics and mathematically similar to the continuity equation.

Proof

Suppose we have some charge and current configuration which, at time **t**, produces field $\vec{E}$ and $\vec{B}$. In the next instant **dt,** the charge move around a bit.

∴ The work done on an element of charge **dq** in time **dt** is

$$dw = \overrightarrow{F} \cdot \overrightarrow{dl} = dq\,[\vec{E} + (\vec{v} \times \overrightarrow{B})] \cdot \vec{v}\,dt$$

$$= \vec{E} \cdot \vec{v}\,dq\,dt \qquad\qquad [Since\ (\vec{v} \times B) \cdot \vec{v} = 0]$$

So the total work done on all charges in some volume **V per sec** is given by:

$$\frac{dW}{dt} = \int_V (\vec{E} \cdot \vec{v})\,dq = \int_V (\vec{E} \cdot \vec{v})\,\rho dV$$

Since dq = charge density × volume element = ρdV

$\rho\overrightarrow{v} = \vec{J}$ = **Current Density Vector**

$$\therefore \frac{dW}{dt} = \int_V (\vec{E} \cdot \vec{J})\,dV \qquad \ldots\ldots\ldots\ldots (1)$$

$$\left(\because \rho\vec{v} = \rho\frac{dl}{dt} = \rho\frac{dl\,ds}{dt\,ds} = \frac{dq}{dt}/ds = J\right.$$

Now from Maxwell's 4th equations we have,

$$\vec{\nabla} \times \vec{B} = \mu \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) = \left(\mu \vec{J} + \mu\varepsilon \frac{\partial \vec{E}}{\partial t}\right)$$

μ = permeability and ε = permittivity of the medium

$$\vec{J} = \frac{\vec{\nabla} \times \vec{B}}{\mu} - \varepsilon \frac{\partial \vec{E}}{\partial t}$$

$$\vec{E} \cdot \vec{J} = \frac{\vec{E} \cdot (\vec{\nabla} \times \vec{B})}{\mu} - \varepsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} \qquad \dots\dots\dots\dots\dots\dots (2)$$

Now, $\vec{\nabla} \cdot (\vec{E} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{E}) - \vec{E} \cdot (\vec{\nabla} \times \vec{B})$

$$\vec{E} \cdot (\vec{\nabla} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{E}) - \vec{\nabla} \cdot (\vec{E} \times \vec{B})$$

Since $\frac{\partial}{\partial t}(\vec{B} \cdot \vec{B}) = \vec{B} \cdot \frac{\partial \vec{B}}{\partial t} + \vec{B} \cdot \frac{\partial \vec{B}}{\partial t} \Rightarrow \vec{B} \cdot \frac{\partial \vec{B}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t} B^2$

Now from Maxwell's 3rd equations we have, $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$\therefore \vec{E} \cdot (\vec{\nabla} \times \vec{B}) = -\vec{B} \cdot \frac{\partial \vec{B}}{\partial t} - \vec{\nabla} \cdot (\vec{E} \times \vec{B})$$

From eqn (2) we get,

$$\vec{E} \cdot \vec{J} = \frac{1}{\mu}[-\vec{B} \cdot \frac{\partial \vec{B}}{\partial t} - \vec{\nabla} \cdot (\vec{E} \times \vec{B})] - \varepsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t}$$

Since $\vec{B} \cdot \frac{\partial \vec{B}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}(B^2)$ and $\vec{E} \cdot \frac{\partial \vec{E}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}(E^2)$, therefore

$$\int_V \vec{E} \cdot \vec{J} \, dV = -\frac{1}{2}\frac{d}{dt}\int_V \left(\varepsilon E^2 + \frac{B^2}{\mu}\right) dV - \frac{1}{\mu}\int_V \vec{\nabla} \cdot (\vec{E} \times \vec{B}) \, dV \quad \text{........ (3)}$$

Putting eqn (3) in eqn (1) and applying the divergence theorem to the second term, we have

$$\frac{dW}{dt} = -\frac{d}{dt}\int_V \frac{1}{2}\left(\varepsilon E^2 + \frac{B^2}{\mu}\right) dV - \frac{1}{\mu}\int_V \vec{\nabla} \cdot (\vec{E} \times \vec{B}) \, dV$$

Since $\oint_S (\vec{E} \times \vec{B}) \cdot d\vec{a} = \int_V \nabla \cdot (\vec{E} \times \vec{B})\, dV$

$$\frac{dW}{dt} = -\frac{d}{dt}\int_V \frac{1}{2}\left(\varepsilon E^2 + \frac{B^2}{\mu}\right) dV - \frac{1}{\mu}\oint_A \left(\vec{E} \times \vec{B}\right) \cdot \overrightarrow{da} \quad \text{.........(4)}$$

$(Where\ A\ is\ the\ Surface\ Bounding\ volume\ V)$ Equation (4) gives Poynting's theorem.

From eqn (3) we have

$$-\frac{1}{2}\frac{\partial}{\partial t}\int_V \left(\varepsilon E^2 + \frac{B^2}{\mu}\right) dV = \int_V \vec{E} \cdot \vec{J}\, dV + \frac{1}{\mu}\int_V \vec{\nabla} \cdot \left(\vec{E} \times \vec{B}\right) dV \quad \text{.........(5)}$$

This term $-\frac{1}{2}\frac{\partial}{\partial t}\int_V \left(\varepsilon E^2 + \frac{B^2}{\mu}\right) dV$ represents the rate decreasing of total electromagnetic energy in volume V.

The second term (R.H.S.) in equation (5) represents the rate at which energy is carried out of the volume V, across its boundary surface by e.m. fields. Therefore, we write: $\frac{1}{\mu}\oint_A (\vec{E}\times\vec{B})\cdot\overrightarrow{da} = \oint_A \vec{S}\cdot\overrightarrow{da}$

Where, $\vec{S}\cdot\overrightarrow{da}$ is electromagnetic energy per unit time passing through the infinitesimal surface $\overrightarrow{da}$ that is the directional energy flux. Therefore

$$\vec{S} = \frac{1}{\mu}(\vec{E}\times\vec{B}) = (\vec{E}\times\vec{H})$$ **is the directional energy flux density or Poynting's Vector.**

It gives the amount of electromagnetic energy crossing out normally per unit area per unit time.

- **Poynting's vector** is a quantity describing the magnitude and direction of the flow of energy in electromagnetic waves. It is named after English physicist John Henry Poynting, who introduced it in 1884.
- The SI unit of the Poynting's vector is watt per square metre (W/m^2).

$$\vec{H} = \frac{\vec{B}}{\mu} \Rightarrow \vec{B} = \mu\vec{H}$$

ONLINE LIVE LECTURES

PHYSICS (KAS 101)

Kamla Nehru Institute of Technology Sultanpur
Uttar Pradesh

Unit-3

-: Quantum Mechanics :-

Wave - Particle Duality :-

Every moving particle has wave nature. This concept is given by D' broglie so associates waves with moving particle is called D' broglie waves or matter waves.

According to D' broglie, wave length of matter waves is

$$\lambda = \frac{h}{p}$$

where p is linear momentum of the particle

⇒ If particle is non-relativistic i.e $v << c$
then

$$p = mv$$

so

$$\lambda = \frac{h}{mv}$$

⇒ If particle is relativistic
then

$$p = \gamma m_0 v$$

so

$$\lambda = \frac{h}{\gamma m_0 v}$$

* charge particle in pot. V .

$$p = \sqrt{2mE_k} = \sqrt{2mqV}$$

so

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

for e^-,

so

$$\lambda = \frac{h}{\sqrt{2meV}}$$

* In terms of K.E.

$$p = \frac{\sqrt{E_k(E_k + 2m_0c^2)}}{c}$$

So

$$\lambda = \frac{hc}{\sqrt{E_k(E_k + 2m_0c^2)}}$$

if there is a charge particle in Pot. V,

$$\lambda = \frac{hc}{\sqrt{qV(qV + 2m_0c^2)}}$$

Key idea,

$$\lambda = \frac{h}{p}$$

Rel. particle

$$\lambda = \frac{h}{\gamma m_0 v}$$

↓ in terms of K.E.

$$\lambda = \frac{hc}{\sqrt{E_K(E_K + 2m_0c^2)}}$$

non-rel. particle

$$\lambda = \frac{h}{m_0 v}$$

↓ in term of K.E.

$$\lambda = \frac{h}{\sqrt{2mE_K}}$$

For charge particle

$$\lambda = \frac{hc}{\sqrt{qV(qV + 2m_0c^2)}}$$

$$\lambda = \frac{h}{\sqrt{2m_0 qV}}$$

* For gas particle,

$$E_k = \frac{3}{2} k_B T$$

$$\lambda = \frac{hc}{\sqrt{\frac{3}{2}k_BT\left(\frac{3}{2}k_BT + 2m_0c^2\right)}}$$

$$\lambda = \frac{h}{\sqrt{2m_0 \frac{3}{2} k_B T}}$$

1. Calculate de-Broglie wavelength of an alpha particle accelerated through a potential difference of 200 V.
2. A proton is moving with a speed of 2×10^8 m/s. Find the wavelength of the matter wave associated with it.
3. Calculate the wavelength of an electron that has been accelerated in a particle accelerator through a potential difference of 100 V.
4. Calculate de-Broglie wavelength of neutron of energy 12.8 MeV.
5. Calculate the kinetic energy of an electron if its de-Broglie wavelength equals the wavelength of the yellow line of sodium (5796 Å).
6. The kinetic energy of an electron is 4.55×10^{-25} J. Calculate velocity, momentum and wavelength of electron.

Sol^n: 1 $q = 2e$, $V = 200V$ then $\lambda = \frac{h}{\sqrt{2mqV}}$

$m = 4m_p$

Sol^n:- 2 $v = 2 \times 10^8$ m/s, $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \ldots$ $m_0 = m_p$

$$\lambda = \frac{h}{\gamma m_0 v}$$

Sol^n 4 :- rest mass of neutron

$$m_0 = 939 \text{ MeV}/c^2$$

$$E_K = 12.8 \text{ MeV}$$

$$\lambda = \frac{hc}{\sqrt{E_K(E_K + 2m_0c^2)}} = \text{- - - - - - meter}$$

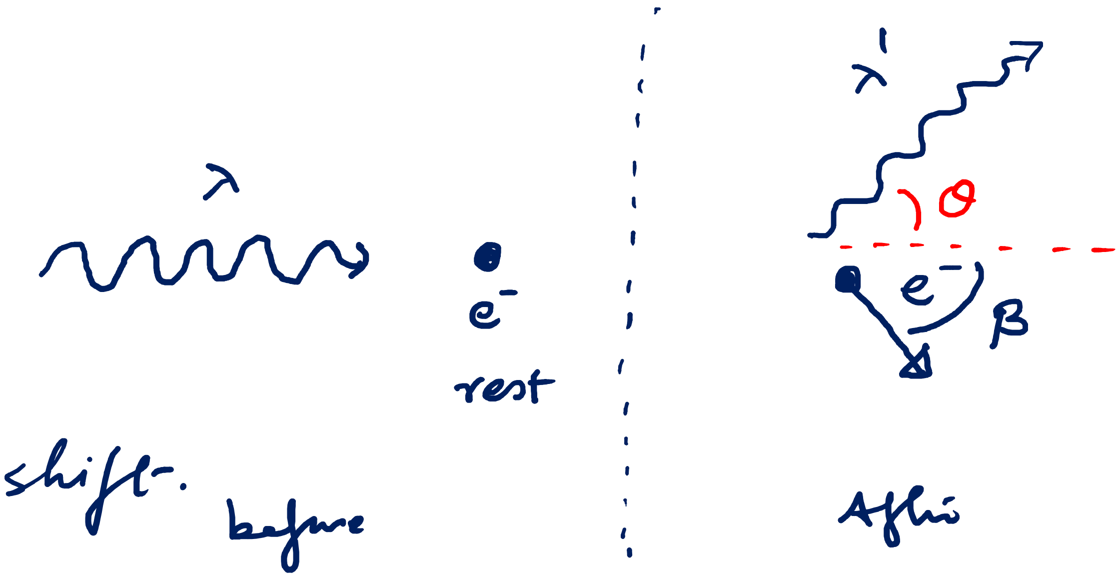
$$p = \frac{\sqrt{K(K + 2m_0c^2)}}{c}$$

$$= \sqrt{12.8(12.8 + 939 \times 2)}$$

$$p = 155.56 \text{ MeV}/c$$

Compton's effect :-

$$\lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$$



This is called Compton's shift.

proof

Energy conservation law,

$$E_{before} = E_{After}$$

$$\frac{hc}{\lambda} + m_0c^2 = \frac{hc}{\lambda'} + m_0c^2 + K$$

$$\frac{hc}{\lambda} = \frac{hc}{\lambda'} + K \quad \text{--- (1)}$$

momentum conservation law,

$$\vec{p}_{before} = \vec{p}_{After}$$

$\Rightarrow$ Along x-axis,

$$(p_x)_{before} = (p_x)_{After}$$

$$\frac{h}{\lambda} + 0 = \frac{h}{\lambda'}\cos\theta + p_e\cos\beta$$

$$\frac{h}{\lambda} - \frac{h}{\lambda'}\cos\theta = p_e\cos\beta \quad \text{———} (2)$$

$\Rightarrow$ Along y-axis

$$(p_y)_{before} = (p_y)_{After}$$

$$0 + 0 = \frac{h}{\lambda'}\sin\theta - p_e\sin\beta$$

$$\frac{h}{\lambda'}\sin\theta = p_e\sin\beta \quad \text{———} (3)$$

$[eq^n ②]^2 + [eq^n ③]^2$

$$\left(\frac{h}{\lambda} - \frac{h}{\lambda'}\cos\theta\right)^2 + \left(\frac{h}{\lambda'}\sin\theta\right)^2 = p_e^2$$

$$\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - 2\frac{h^2}{\lambda\lambda'}\cos\theta = \frac{E^2 - m_0^2c^4}{c^2}$$

$$\left(\because \; E^2 = p_e^2c^2 + m_0^2c^4\right)$$

$$\frac{h^2c^2}{\lambda^2} + \frac{h^2c^2}{\lambda'^2} - 2\frac{h^2c^2}{\lambda\lambda'}\cos\theta = (E + m_0c^2)(E - m_0c^2)$$

$$= (K + 2m_0c^2)K$$

$$\left(\because \; E = K + m_0c^2\right)$$

$$\frac{h^2c^2}{\lambda^2} + \frac{h^2c^2}{\lambda'^2} - 2\frac{h^2c^2}{\lambda\lambda'}\cos\theta = \left(\frac{hc}{\lambda} - \frac{hc}{\lambda'}\right)^2 + 2m_oc^2\left(\frac{hc}{\lambda} - \frac{hc}{\lambda'}\right)$$

(Using eq^n ①)

$$-2\frac{h^2c^2}{\lambda\lambda'}\cos\theta = -2\frac{h^2c^2}{\lambda\lambda'} + 2m_oc^2 \cdot hc \; \frac{\lambda'-\lambda}{\lambda\lambda'}$$

$$-h\cos\theta = -h + m_oc(\lambda'-\lambda)$$

$$\boxed{\lambda' - \lambda = \frac{h}{m_oc}(1-\cos\theta)}$$

$$\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

Here $\frac{h}{m_0 c} = \lambda_c$ (Compton wavelength) $= 0.024\ \text{Å}$

Case I :- if $\theta = 0^\circ$

$$(\Delta\lambda)_{min} = 0$$

Case II :- if $\theta = 90^\circ$

$$\Delta\lambda = \frac{h}{m_0 c} = 0.024\ \text{Å}$$

Case III :- if $\theta = 180^\circ$

$$(\Delta\lambda)_{max} = 2\frac{h}{m_0 c} = 0.048\ \text{Å}$$

Note:-

Wavelength of light-

$\lambda = 3800\,\text{Å} - 7800\,\text{Å}$

wave length of x-Ray

$\lambda = 1$ to $100\,\text{Å}$ = few Ås

Compton effect is not observable for visible radiation.

Unit - 1 to Unit - 5

all units.

POYNTING'S THEOREM

(Work- energy theorem)

- In electrodynamics Poynting's theorem is a statement of conservation of energy for the electromagnetic field.
- Poynting's theorem is analogous to the work-energy theorem in classical mechanics and mathematically similar to the continuity equation.

Proof

Suppose we have some charge and current configuration which, at time **t**, produces field $\vec{E}$ and $\vec{B}$. In the next instant **dt,** the charge move around a bit.

∴ The work done on an element of charge **dq** in time **dt** is

$$dw = \vec{F} \cdot \vec{dl} = dq\,[\vec{E} + (\vec{v} \times \vec{B})] \cdot \vec{v}dt$$

$$= \vec{E} \cdot \vec{v}\, dq\, dt \qquad [Since\ (\vec{v} \times B) \cdot \vec{v} = 0]$$

So the total work done on all charges in some volume **V per sec** is given by:

$$\frac{dW}{dt} = \int_V (\vec{E} \cdot \vec{v})\, dq = \int_V (\vec{E} \cdot \vec{v})\, \rho dV$$

$$Since\ dq = charge\ density \times volume\ element = \rho dV$$

$\rho\vec{v} = \vec{J}$ = **Current Density Vector**

$$\therefore \frac{dW}{dt} = \int_V (\vec{E} \cdot \vec{J})\, dV \qquad \ldots\ldots\ldots\ldots (1)$$

$$(\because \rho\vec{v} = \rho \frac{dl}{dt} = \rho \frac{dl\,ds}{dt\,ds} = \frac{dq}{dt} / ds = J$$

Now from Maxwell's 4th equations we have,

$$\vec{\nabla} \times \vec{B} = \mu \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) = \left(\mu \vec{J} + \mu \varepsilon \frac{\partial \vec{E}}{\partial t}\right)$$

μ = permeability and ε = permittivity of the medium

$$\vec{J} = \frac{\vec{\nabla} \times \vec{B}}{\mu} - \varepsilon \frac{\partial \vec{E}}{\partial t}$$

$$\vec{E} \cdot \vec{J} = \frac{\vec{E} \cdot (\vec{\nabla} \times \vec{B})}{\mu} - \varepsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} \qquad \text{.................. (2)}$$

Now, $\vec{\nabla} \cdot (\vec{E} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{E}) - \vec{E} \cdot (\vec{\nabla} \times \vec{B})$

$$\vec{E} \cdot (\vec{\nabla} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{E}) - \vec{\nabla} \cdot (\vec{E} \times \vec{B})$$

Since $\frac{\partial}{\partial t}(\vec{B} \cdot \vec{B}) = \vec{B} \cdot \frac{\partial \vec{B}}{\partial t} + \vec{B} \cdot \frac{\partial \vec{B}}{\partial t} \Rightarrow \vec{B} \cdot \frac{\partial \vec{B}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t} B^2$

Now from Maxwell's 3rd equations we have, $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$\therefore \vec{E} \cdot (\vec{\nabla} \times \vec{B}) = -\vec{B} \cdot \frac{\partial \vec{B}}{\partial t} - \vec{\nabla} \cdot (\vec{E} \times \vec{B})$$

From eqn (2) we get,

$$\vec{E} \cdot \vec{J} = \frac{1}{\mu}\left[-\vec{B} \cdot \frac{\partial \vec{B}}{\partial t} - \vec{\nabla} \cdot (\vec{E} \times \vec{B})\right] - \varepsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t}$$

Since $\vec{B} \cdot \frac{\partial \vec{B}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}(B^2)$ and $\vec{E} \cdot \frac{\partial \vec{E}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}(E^2)$, therefore

$$\int_V \vec{E} \cdot \vec{J}\, dV = -\frac{1}{2}\frac{d}{dt}\int_V \left(\varepsilon E^2 + \frac{B^2}{\mu}\right) dV - \frac{1}{\mu}\int_V \vec{\nabla} \cdot (\vec{E} \times \vec{B})\, dV \quad \text{........ (3)}$$

Putting eqn (3) in eqn (1) and applying the divergence theorem to the second term, we have

$$\frac{dW}{dt} = -\frac{d}{dt}\int_V \frac{1}{2}\left(\varepsilon E^2 + \frac{B^2}{\mu}\right) dV - \frac{1}{\mu}\int_V \vec{\nabla} \cdot (\vec{E} \times \vec{B})\, dV$$

Since $\oint_S (\vec{E} \times \vec{B}) \cdot d\vec{a} = \int_V \nabla \cdot (\vec{E} \times \vec{B})\, dV$

$$\frac{dW}{dt} = -\frac{d}{dt}\int_V \frac{1}{2}\left(\varepsilon E^2 + \frac{B^2}{\mu}\right) dV - \frac{1}{\mu}\oint_A \left(\vec{E} \times \vec{B}\right) \cdot \overrightarrow{da} \qquad \text{.........(4)}$$

$(Where\ A\ is\ the\ Surface\ Bounding\ volume\ V)$ Equation (4) gives Poynting's theorem.

From eqn (3) we have

$$-\frac{1}{2}\frac{\partial}{\partial t}\int_V \left(\varepsilon E^2 + \frac{B^2}{\mu}\right) dV = \int_V \vec{E} \cdot \vec{J}\, dV + \frac{1}{\mu}\int_V \vec{\nabla} \cdot \left(\vec{E} \times \vec{B}\right) dV \qquad \text{.........(5)}$$

This term $-\frac{1}{2}\frac{\partial}{\partial t}\int_V \left(\varepsilon E^2 + \frac{B^2}{\mu}\right) dV$ represents the rate decreasing of total electromagnetic energy in volume V.

The second term (R.H.S.) in equation (5) represents the rate at which energy is carried out of the volume V, across its boundary surface by e.m. fields. Therefore, we write: $\frac{1}{\mu}\oint_A (\vec{E}\times\vec{B})\cdot\overrightarrow{da} = \oint_A \vec{S}\cdot\overrightarrow{da}$

Where, $\vec{S}\cdot\overrightarrow{da}$ is electromagnetic energy per unit time passing through the infinitesimal surface $\overrightarrow{da}$ that is the directional energy flux. Therefore

$$\vec{S} = \frac{1}{\mu}(\vec{E}\times\vec{B}) = (\vec{E}\times\vec{H})$$ **is the directional energy flux density or Poynting's Vector.**

It gives the amount of electromagnetic energy crossing out normally per unit area per unit time.

- **Poynting's vector** is a quantity describing the magnitude and direction of the flow of energy in electromagnetic waves. It is named after English physicist John Henry Poynting, who introduced it in 1884.
- The SI unit of the Poynting's vector is watt per square metre (W/m^2).

$$\vec{H} = \frac{\vec{B}}{\mu} \Rightarrow \boxed{\vec{B} = \mu\vec{H}}$$

ONLINE LIVE LECTURES

PHYSICS (KAS 101)

Kamla Nehru Institute of Technology Sultanpur
Uttar Pradesh

Unit-3

-: Quantum Mechanics :-

Wave - Particle Duality :-

Every moving particle has wave nature. This concept is given by D' broglie so associates waves with moving particle is called D' broglie waves or matter waves.

According to D' broglie, wave length of matter waves is

$$\lambda = \frac{h}{p}$$

where p is linear momentum of the particle

⇒ If particle is non-relativistic i.e $v << c$
then

$$p = mv$$

so

$$\lambda = \frac{h}{mv}$$

⇒ If particle is relativistic
then

$$p = \gamma m_0 v$$

so

$$\lambda = \frac{h}{\gamma m_0 v}$$

* charge particle in pot. V.

$$p = \sqrt{2mE_k} = \sqrt{2mqV}$$

so

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

for e^-,

so

$$\lambda = \frac{h}{\sqrt{2meV}}$$

* In terms of K.E.

$$p = \frac{\sqrt{E_k(E_k + 2m_0c^2)}}{c}$$

so

$$\lambda = \frac{hc}{\sqrt{E_K(E_K + 2m_0c^2)}}$$

if there is a charge particle in Pot. V,

$$\lambda = \frac{hc}{\sqrt{qV(qV + 2m_0c^2)}}$$

Key idea,

$$\boxed{\lambda = \frac{h}{p}}$$

Rel. particle →

$$\lambda = \frac{h}{\gamma m_0 v}$$

↓ in terms of K.E.

$$\lambda = \frac{hc}{\sqrt{E_K(E_K + 2m_0c^2)}}$$

non-rel. particle →

$$\lambda = \frac{h}{m_0 v}$$

↓ in term of K.E.

$$\lambda = \frac{h}{\sqrt{2mE_K}}$$

↓ for charge particle

$$\lambda = \frac{hc}{\sqrt{qV(qV + 2m_0c^2)}}$$

↓

$$\lambda = \frac{h}{\sqrt{2m_0 qV}}$$

* for gas particle,

$$E_k = \frac{3}{2}k_B T$$

↓

$$\lambda = \frac{hc}{\sqrt{\frac{3}{2}k_B T\left(\frac{3}{2}k_B T + 2m_0c^2\right)}}$$

↓

$$\lambda = \frac{h}{\sqrt{2m_0 \frac{3}{2}k_B T}}$$

1. Calculate de-Broglie wavelength of an alpha particle accelerated through a potential difference of 200 V.
2. A proton is moving with a speed of 2×10^8 m/s. Find the wavelength of the matter wave associated with it.
3. Calculate the wavelength of an electron that has been accelerated in a particle accelerator through a potential difference of 100 V.
4. Calculate de-Broglie wavelength of neutron of energy 12.8 MeV.
5. Calculate the kinetic energy of an electron if its de-Broglie wavelength equals the wavelength of the yellow line of sodium (5796 Å).
6. The kinetic energy of an electron is 4.55×10^{-25} J. Calculate velocity, momentum and wavelength of electron.

Sol^n: 1 $q = 2e$, $V = 200V$ then $\lambda = \frac{h}{\sqrt{2mqV}}$

$m = 4m_p$

Sol^n:- 2 $v = 2 \times 10^8$ m/s, $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \dots$ $m_0 = m_p$

$$\lambda = \frac{h}{\gamma m_0 v}$$

Sol^n 4 :- rest mass of neutron

$$m_0 = 939 \text{ MeV}/c^2$$

$$E_K = 12.8 \text{ MeV}$$

$$\lambda = \frac{hc}{\sqrt{E_K(E_K + 2m_0c^2)}} = \text{- - - - - meter}$$

$$p = \frac{\sqrt{K(K + 2m_0c^2)}}{c}$$

$$= \sqrt{12.8(12.8 + 939 \times 2)}$$

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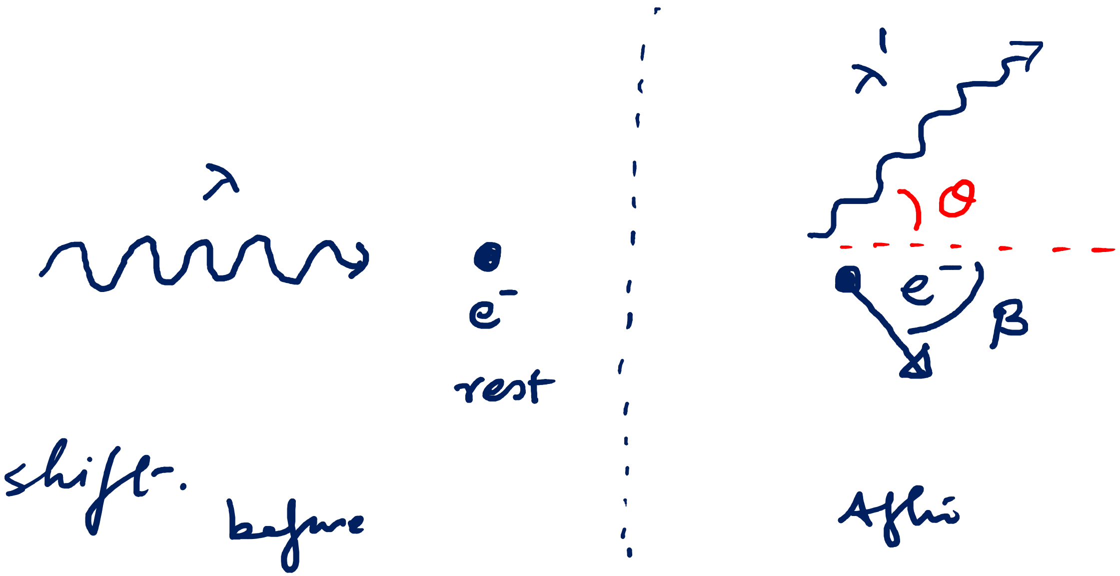
$$\boxed{\lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)}$$



This is called Compton's shift.

proof

Energy conservation law,

$$E_{before} = E_{After}$$

$$\frac{hc}{\lambda} + \cancel{m_0 c^2} = \frac{hc}{\lambda'} + \cancel{m_0 c^2} + K$$

$$\frac{hc}{\lambda} = \frac{hc}{\lambda'} + K \qquad \text{— (1)}$$

momentum conservation law,

$$\vec{p}_{before} = \vec{p}_{After}$$

$\Rightarrow$ Along x-axis,

$$(p_x)_{before} = (p_x)_{After}$$

$$\frac{h}{\lambda} + 0 = \frac{h}{\lambda'}\cos\theta + p_e \cos\beta$$

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$\Rightarrow$ Along y-axis

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$[eq^n ②]^2 + [eq^n ③]^2$

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$$\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - 2\frac{h^2}{\lambda\lambda'}\cos\theta = \frac{E^2 - m_0^2c^4}{c^2}$$

$$(\because \; E^2 = p_e^2c^2 + m_0^2c^4)$$

$$\frac{h^2c^2}{\lambda^2} + \frac{h^2c^2}{\lambda'^2} - 2\frac{h^2c^2}{\lambda\lambda'}\cos\theta = (E + m_0c^2)(E - m_0c^2)$$

$$= (K + 2m_0c^2)K$$

$$(\because \; E = K + m_0c^2)$$

$$\frac{h^2c^2}{\lambda^2} + \frac{h^2c^2}{\lambda'^2} - 2\frac{h^2c^2}{\lambda\lambda'}\cos\theta = \left(\frac{hc}{\lambda} - \frac{hc}{\lambda'}\right)^2 + 2m_oc^2\left(\frac{hc}{\lambda} - \frac{hc}{\lambda'}\right)$$

(Using eq^n ①)

$$-\cancel{2}\frac{h^2c^2}{\lambda\lambda'}\cos\theta = -\cancel{2}\frac{h^2c^2}{\lambda\lambda'} + \cancel{2}m_oc^2 \cdot hc\,\frac{\lambda'-\lambda}{\lambda\lambda'}$$

$$-h\cos\theta = -h + m_oc(\lambda'-\lambda)$$

$$\boxed{\lambda'-\lambda = \frac{h}{m_oc}(1-\cos\theta)}$$

$$\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

Here $\frac{h}{m_0 c} = \lambda_c$ (Compton wavelength) $= 0.024\ \text{Å}$

Case I :- if $\theta = 0°$

$$(\Delta\lambda)_{min} = 0$$

Case II :- if $\theta = 90°$

$$\Delta\lambda = \frac{h}{m_0 c} = 0.024\ \text{Å}$$

Case III :- if $\theta = 180°$

$$(\Delta\lambda)_{max} = 2\frac{h}{m_0 c} = 0.048\ \text{Å}$$

Note:-

Wavelength of light-

$\lambda = 3800\,\text{Å} — 7800\,\text{Å}$

wave length of X-Ray

$\lambda = 1$ to $100\,\text{Å}$ = few Ås

Compton effect is not observable for visible radiation.

Wave funtion :-

$|\psi|^2$ represents the probability density.

$|\psi|^2 dz$ probability

Properties of well behaved wave funtion:-

It should be ⇒ single valued

⇒ Coninuous

⇒ differentiable means $\frac{\partial \psi}{\partial x}$, $\frac{\partial \psi}{\partial y}$ .. should be continuous

⇒ As $x, y, z \rightarrow \pm\infty$, $\psi \rightarrow 0$

Schroedinger's wave eqⁿ:-

M. Imp

$$\Rightarrow \nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

$$\Rightarrow H\psi = E\psi$$

or

$$\nabla^2\psi = -\frac{2m}{\hbar^2}\left[E\psi - V\psi\right]$$

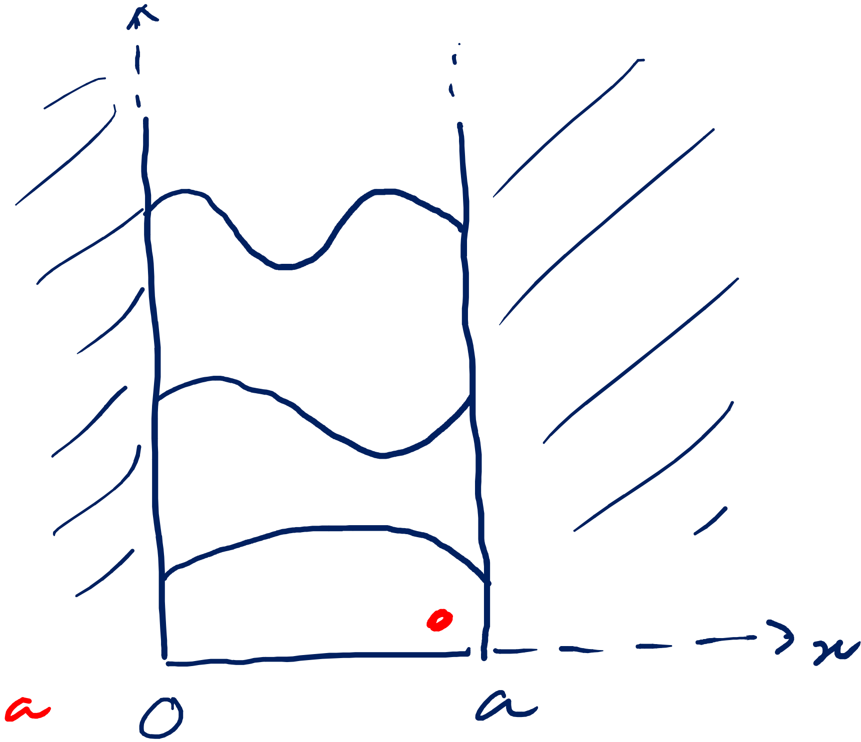
$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = E\psi$$

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = \frac{\hbar}{i}\frac{\partial\psi}{\partial t}$$

Application of Schrodinger's eq^n :-

⇒ Particle in a pot. box :-

$$V(x) = \begin{cases} 0 & 0 < x < a \\ \infty & \text{otherwise} \end{cases}$$



$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(E - 0)\psi = 0$$

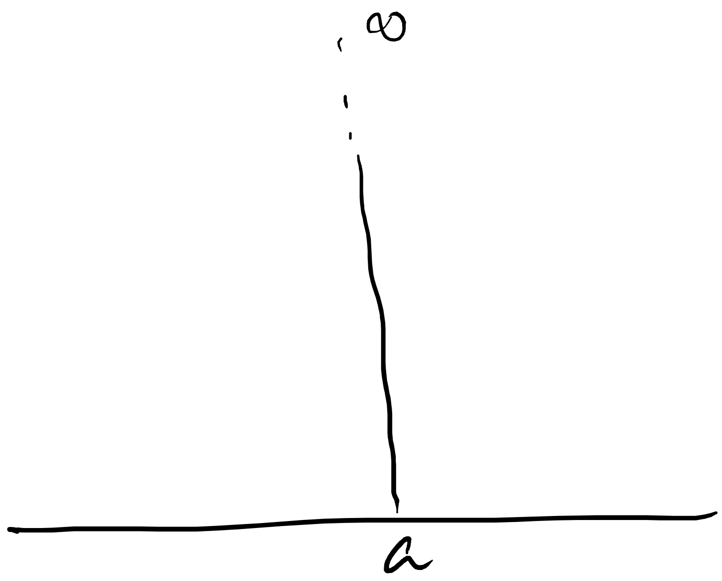
For $0 < x < a$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2mE}{\hbar^2}\psi = 0 \Rightarrow \frac{\partial^2 \psi}{\partial x^2} + \alpha^2 \psi = 0$$

or $D^2 + \alpha^2 = 0$

Dirac - Delta function :-

$$\delta(x) = \begin{cases} \infty & \text{at } x = a \\ 0 & \text{otherwise.} \end{cases}$$



$D = \pm i\alpha$

Sol^n of diff eq^n.

$\psi(x) = A \sin \alpha x + B \cos \alpha x$

using boundary condition

$$\psi(x) = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$$

Here $n = 1, 2, 3, \ldots$

$$\sqrt{\frac{2mE}{\hbar^2}}\, a = n\pi$$

$$\frac{2mE}{\hbar^2}\, a^2 = n^2\pi^2$$

$$\boxed{E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}}$$

$n = 1$

$$E_1 = \frac{\pi^2\hbar^2}{2ma^2}$$ (ground energy)

$n = 2$ $\quad E_2 = 4E_1$ — — —

⇒ Plank's radiation formula

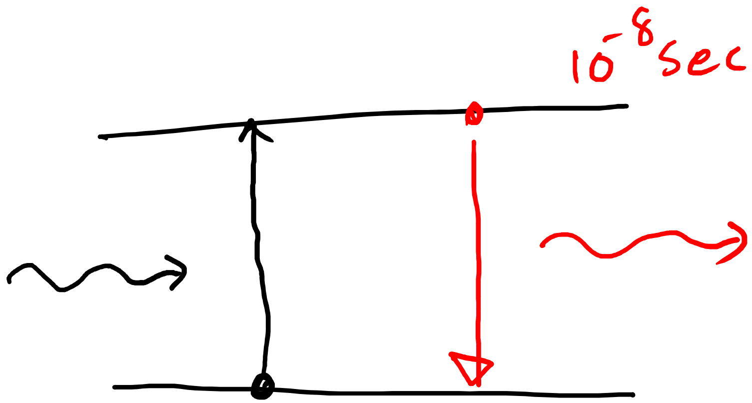
- Reighley Jeen's law
- Wein's law.

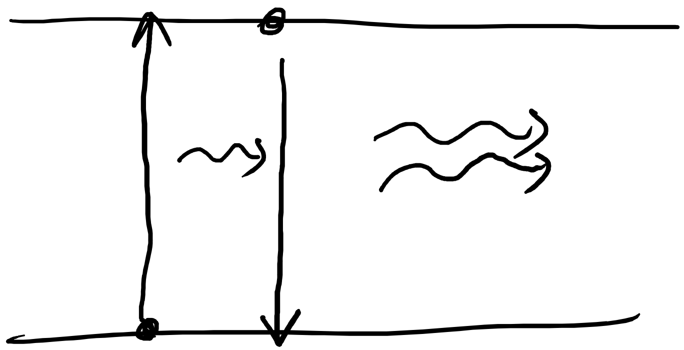
Unit- 5

* LASER:-

⇒ Full form.

⇒ Stimulated & spontaneous emission





⇒ population inversion

⇒ pumping

⇒ Ruby laser (I^{st} laser)

Optical fiber :-

⇒ principle

⇒ TIR

⇒ Construction

Unit - 4

⇒ Interference

⇒ Thin film

⇒ Newton's Ring.